2 Lesson 2
RATIO, PROPORTION, AND PERCENT

Thinking KAP

A summer camp has *x* number of people at the camp. The ratio of counselors to campers is 1:8.

Can there be a total of 27 people at the summer camp? yes

Can there be a total of 36 people at the summer camp? yes

Can there be a total of 45 people at the summer camp? yes

Can there be a total of 37 people at the summer camp? no

Why not?

The total number of people at the camp needs to be a multiple of 9 to account for the 1:8 ratio of counselors to campers.

If there are 54 people at the camp, how many of them are campers? 48

How do you know?

(sample answer) The number of people at the camp needs to fit the ratio of 1 counselor for every 8 campers. The ratio of campers to total people at the camp is 8:9 so if there are 54 people, 48 of them are campers, allowing for 6 counselors.

Thinking KAP

approx. 4 min.

Delivery

Have students complete the Thinking KAP activity independently.
Alternatively, you may choose to have students complete this activity in pairs.

Lead a discussion about ratios.
Have students share their responses to the activity. The discussion should be guided by student responses, but you may wish to address the following points:

- The ratio 1:8 means that the smallest total number of people that can possibly be at the camp is 9 (1 counselor and 8 campers). Point out that the first three questions involve numbers that are all multiples of 9. Emphasize that for every one counselor, there are eight campers; therefore, the total number of people will always be a multiple of 9.
- Help struggling students by encouraging them to draw a diagram to represent the ratio. Remind students to use simple drawings in their diagram, and not to focus on the visual details of campers and counselors. For example, if students represent campers and counselors as circles and squares, a 1:8 ratio will result in 9 total shapes.

Instruction

approx. 6 min.

Delivery

Read the introductory text with students.

- Help students make the connection between the Thinking KAP activity and the instruction by explaining that, in the Thinking KAP, students used a part-to-part ratio to determine whether or not a particular value could represent the whole.
- Review the three forms in which a ratio can be written. Remind students that, since a ratio can be written as a fraction, the ratio stays the same when written as any value equivalent to that fraction.
- Explain that, when a part-to-part ratio is given, students can use the size of each part to determine the part-to-whole ratio.

Introduce the strategy: Ratio Charts.

- Read the steps of creating Ratio Charts with students. Explain that using a Ratio Chart is part of Step 2 of the 4-Step Method for Problem Solving.
- Emphasize that Ratio Charts can help students organize the given information in a problem, to see the proportional relationships more clearly, and to use that information to solve for an unknown value.

Guide students through the Try It Out exercise.

- Review the text and Ratio Chart with students.
- Have students describe the three proportional relationships that are visible in the Ratio Chart and discuss student responses aloud.
- If students struggle to see that the ratios are equivalent, use smaller numbers to represent the relationship between boys and girls. Have students identify the number of boys that would be in the class if there were 2 girls, then 3 girls, etc.

Content Background

Two ratios are equal (in proportion) if and only if the cross-products are equal. Algebraically, this is represented as follows:

$$\frac{a}{b} = \frac{c}{d} \qquad a \cdot d = b \cdot c$$

Instruction

Ratios

A ratio is a way of expressing the relative sizes of two quantities. Ratios can be written in three different forms: with words, with a colon (:), or as a fraction.

WORD FORM	COLON FORM	FRACTION FORM
1 to 3	1:3	$\frac{1}{3}$

A ratio can compare a part to a part, a part to a whole, or a whole to a part. For example, the ratios above may represent the ratio of 1 boy to 3 girls in a class (part to part), or 1 question correct out of 3 questions total (part to whole).

Inside the SAT!

On the SAT, ratio problems may be written in a way that tricks you into setting up the wrong ratio. Be sure to read the problem carefully.

Ratio Charts

On the SAT, Ratio Charts can help you solve problems that involve ratios, proportions, or percents.

Ratio Charts

- Write a known ratio in the Ratio 1 column.
- Label the units or the figures on the left side of the Ratio Chart.
- Use the units as a guide to fill in the missing information in the Ratio 2 column.

Once you have filled in a Ratio Chart to display and analyze the important information in a problem, you can use the number relationships to find the unknown number.

TRY IT OUT **Find three number relationships in the Ratio Chart below.**

In the chart below, the first column represents the ratio of girls to boys in a class of 24 students. The second column represents the actual number of girls and boys in the class.

	Ratio 1	Ratio 2
Girls	1	8
Boys	2	16

Describe three number relationships you see in the Ratio Chart above.

1. *In each column, the top number is half of the bottom number.*
2. *In each row, the second number is 8 times the first number.*
3. *The cross-products are equal (i.e., $1 \times 16 = 16$ and $2 \times 8 = 16$).*

REMEMBER

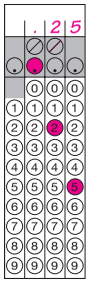
- Two ratios are considered proportional if their cross-products are equal.

Watch the Units

When you create a Ratio Chart, make sure that the units are the same in both ratios. This may mean converting between units before creating a Ratio Chart (like changing yards per second to feet per second). Identify the ratio given in the problem, and notice whether the units in the desired ratio are the same. If not, you'll have to convert the units in the first ratio.

TRY IT OUT **Use a Ratio Chart to solve the problem below.**

1. If an object travels at 5 feet per minute, how many feet does it travel in 3 seconds?



Other answers include: $\frac{1}{4}$

What rate are you provided with in the problem? 5 feet / 1 minute

Rewrite the first ratio using seconds instead of minutes. 5 feet / 60 seconds

Complete the Ratio Chart below using a variable to represent your unknown value.

	Ratio 1	Ratio 2
Feet	5	x
Seconds	60	3

Use any number relationship to solve the problem.

(sample answer) $\frac{5}{60} = \frac{x}{3}$

$5(3) = 60x$

$15 = 60x$

$0.25 = \frac{1}{4} = x$

Inside the SAT!

Many questions that involve rates require you to convert time (for example, from minutes to seconds). Pay close attention to the time units included in the problem.

REMEMBER

- There are three number relationships in a Ratio Chart. Use whichever one makes it easiest to solve the problem.

Instruction

approx. 6 min.

Delivery

Read the introductory text with students.

- Explain that ratios can be compared only when they describe quantities of the same units. For example, if one ratio compares feet per minutes, it cannot be compared to a second ratio in yards per minutes. One ratio must be first converted to the units of the other.
- Explain that a Ratio Chart should reflect the units of measurement needed to answer the problem. This may not be the units of measurement provided in the first ratio. For example, students may be asked a question in which the answer needs to be in yards per minute and they are given a ratio in feet per minute. Students must convert the given ratio in feet per minute to feet per second before entering these values into the Ratio Chart.

Guide students through the Try It Out exercise.
Work through the problem with students.

- Have students underline the units of the ratio provided.
- Emphasize that the units given in the problem are different from the units needed for the answer. Explain that students will need to determine the rate in feet per second, and use this ratio in the Ratio Chart.
- Explain that there is more than one way to find the unknown number in the Ratio Chart. Students could solve a proportion, as shown in the answer key. However, they could also apply other facts they know about equivalent ratios to solve the problem. For example, the number of feet is 1/12 the number of seconds. Since $3 \div 12 = 3/12$, or 1/4, the object travels 1/4 foot in 3 seconds.

Instruction

approx. 6 min.

Delivery

Read the introductory text with students.

- Explain that a percent is a special type of ratio: the ratio of a number to 100. Remind students that when a percent is written as a fraction, the numerator is the part and the denominator is the whole in a part-to-whole ratio.
- Explain that while percent problems can be solved by setting up a proportion, it may help save time with calculations to instead convert the percent to a decimal. Students can convert a percent to a decimal by moving the decimal point two places to the left. For example, 37% can be converted to the decimal 0.37.
- Read each clue word and identify its translation. Have students underline the clue words in the examples provided. Emphasize that they are rewriting percent problems as equations.
- Emphasize that percent problems on the SAT will be written in different ways, so students will need a general tool for translating percent problems into equations.

Guide students through the Try It Out exercise.
Work through the problem with students using percent translations. Remind students that in order to convert 20% to a decimal, students can move the decimal point two places to the left.

Percent→Decimal: 20% → 0.20 Decimal→Percent: 0.20 → 20%

- Remind students to check their work. Have students create a Ratio Chart to try to solve the problem a different way. Remind students that in the Ratio Chart, 20% can be rewritten as the part-to-whole ratio $^{20}/_{100}$. Emphasize that students then need to determine whether 85 is the part or the whole in the second ratio. Students should see that 85 is the part, and they are solving for the whole.

	Ratio 1	Ratio 2
Part	20	85
Whole	100	425

Percents

Some problems may ask you to find a percent of a certain number. The easiest way to do this is to multiply that number by the percent, written as a decimal. You can then your calculator to find the answer quickly. Use the percent translation table below to help you understand how to approach percent problems.

What	Is	Of
x	=	·

What is 16% of 20? $x = 0.16 \cdot 20$

20 is 16% of what? $20 = 0.16 \cdot x$

20 is what percent of 16? $\frac{20}{100} = \frac{x}{16}$

Inside the SAT!

When a quantity is increased or decreased by a percentage more than once, you can't simply add or subtract the percentages to get the answer. The first change is a percentage of the starting amount, but the second change is a percentage of the new amount.

TRY IT OUT **Solve the problem below.**

2. 85 is 20 percent of d. What is the value of d?

(A) 17
(B) 105
(C) 170
(D) 345
(E) 425

In the space below, write an equation for the problem, and use the equation to solve for d.

$85 = 0.20d$
$85 \div 0.20 = 0.20d \div 0.20$
$425 = d$

What is the value of d? 425

REMEMBER

- The word percent comes from the Latin *per centum* meaning "out of one hundred."

More Than One

Problems on the SAT often require you to write more than one equation or Ratio Chart in order to find the solution.

TRY IT OUT **Solve the problem below.**

3. If 125 percent of x is 150, what is x percent of 75?

(A) 70
(B) 90
(C) 120
(D) 150
(E) 185

Inside the SAT!

Don't keep your math on the calculator! If you write equations in your test booklet, including values from your calculator, you are less likely to make careless errors.

Write the first equation in this problem. $1.25 \cdot x = 150$

Solve for x using your calculator.

$$1.25 \cdot x = 150$$
$$\frac{1.25 \cdot x}{1.25} = \frac{150}{1.25}$$
$$x = 120$$

Use your value of x to set up the second equation in this problem. $x = 1.20 \cdot 75$

Solve for the missing number using your calculator.

$75 \cdot 1.20 = 90$

Instruction

approx. 6 min.

Delivery

Read the introductory text with students.

- Explain that many of the ratio, proportion, and percent problems on the SAT will require students to create more than one Ratio Chart or write more than one equation.

Guide students through the Try It Out exercise.
Work through the problem with students. Have students begin by writing out the first equation in the problem using the percent translations they learned on the previous page.

- Remind students that computing their equations twice can help them check their answers and avoid careless errors.

⚠ **Students who write 125% as 0.125 instead of 1.25 may be moving the decimal to the front of the number/percent instead of counting the two spaces left.**

Teacher's Note

Remind students that all questions on the SAT can be answered without a calculator. Using a calculator for each question can actually slow students down and distract them from correctly solving the problems.

Independent Practice

approx. 12 min.

Delivery

Have students complete the Independent Practice.
As students work, observe and assist when necessary. Point out that the questions are more difficult at the end of the set than at the beginning, just as they are in an SAT Mathematics section. Redirect students as needed by asking them questions about their work. Effective questions include:

- What are the important clues in this word problem?
- Do you need to set up a Ratio Chart for this problem? If so, what ratio will you use for Ratio 1?
- How can you rewrite this problem as an equation?

1 D Degree of Difficulty: 2

Students can translate the problem into two separate equations. Students first need to solve for y, and then find 70% of y.

$750 = 0.30y$ $\qquad x = 0.70y$

$750 \div 0.30 = 0.30 \div 0.30y$ $\qquad x = 0.70(2{,}500)$

$2{,}500 = y$ $\qquad x = 1{,}750$

Students who chose (E) may have calculated the value of y, but neglected to determine 70% of *y*. Remind students to restate the question in their own words and to make sure they are answering the correct question when they carry out Part 4 by checking their work.

2 C Degree of Difficulty: 2

Students can translate the problem into two separate equations. Students first need to solve for g, and then to determine what percent of 500 is equal to g.

$0.40g = 50$ $\qquad 125 = \frac{x}{100} \cdot 500$

$0.40g \div 0.40 = 50 \div 0.40$ $\qquad 125 = 5x$

$g = 125$ $\qquad 25 = x$

Students who chose (A) may have calculated what percent of 500 is equal to 50. Remind students to check their work, including making sure they answered the question that was asked, after solving the problem.

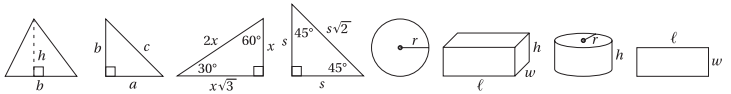
Independent Practice

Use the strategies you know to answer the following questions on your own or with a partner.

Notes

1. Calculator use is permitted.
2. All numbers used are real numbers.
3. Figures are provided for some problems. All figures are drawn to scale and lie in a plane UNLESS otherwise indicated.
4. Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Reference Information



$A = \frac{1}{2}bh$ $\quad c^2 = a^2 + b^2$ $\quad$ Special Right Triangles $\quad A = \pi r^2$ $\quad C = 2\pi r$ $\quad V = \ell wh$ $\quad V = \pi r^2 h$ $\quad A = \ell w$

The number of degrees of arc in a circle is 360.
The sum of the measures in degrees of the angles of a triangle is 180.

1. If 750 is 30 percent of y, what is 70 percent of y?

(A) 525
(B) 1,500
(C) 1,575
(D) 1,750
(E) 2,500

hint *Remember the words "what," "of," and "is" can help in translating the word problem into an equation.*

2. If 40 percent of g is 50, g is what percent of 500?

(A) 10%
(B) 20%
(C) 25%
(D) 30%
(E) 60%

hint *Start by setting up an equation to determine the value of g.*

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3. 40 percent of 210 is the same as $33\frac{1}{3}$ percent of what number?

(A) 70
(B) 84
(C) 252
(D) 294
(E) 630

hint *Begin by rewriting the percent as a fraction.*

4. Jeweler *A* can set an average round-cut diamond in 20 minutes. Jeweler *B* requires 15 minutes to set the same type of diamond. In 8 hours, how many more diamonds can Jeweler *B* set than Jeweler *A*?

(A) 1
(B) 8
(C) 16
(D) 24
(E) 32

hint *Start by using a Ratio Chart to determine how many diamonds Jeweler A can set in 8 hours. Then set up another Ratio Chart to determine how many diamonds Jeweler B can set in 8 hours.*

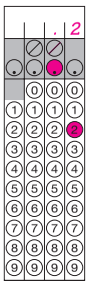
5. The spare change on a dresser is composed of pennies, nickels, and dimes. If the ratio of pennies to nickels is 2:3, and the ratio of pennies to dimes is 3:4, what is the ratio of nickels to dimes?

(A) 9:8
(B) 5:7
(C) 4:5
(D) 3:4
(E) 2:3

hint *Start by making both ratios equivalent by having the same number of pennies.*

6. The ratio of 1.5 to 18 is the same as the ratio of x to 2.4. What is the value of x?

Other answers include: $\frac{1}{5}$



hint *Use a Ratio Chart to set up a proportion. Remember Ratio 1 needs to be equal to Ratio 2 to be proportional.*

Answers

3 C Degree of Difficulty: 3

Students can translate the problem into two separate equations. Students first need to find 40% of 210, before determining the unknown number.

If students chose (B) they may have found 40% of 210, but did not continue on to find the unknown number. Inform students that it is important to read each question carefully in order to answer the correct question.

4 B Degree of Difficulty: 4

Students can first set up a Ratio Chart to determine how many diamonds Jeweler *A* can set in 8 hours. Students should then set up another ratio chart to determine how many diamonds Jeweler *B* can set in the same amount of time.

If students chose (A), they may have calculated the difference between numbers of diamonds each jeweler would set in one hour instead of 8 hours. Emphasize that students were on the right track by making the units of measurement equal in both ratios, but that they always need to check that they are answering the correct question.

5 A Degree of Difficulty: 4

To find the ratio of nickels to dimes, students can first make the two ratios given proportional to each other by having them compare an identical number of pennies. By multiplying the first ratio by 3 to get 6:9 and multiplying the second ratio by 2 to get 6:8, students have ratios with an equal number of pennies in each. Students can then read off the ratio of nickels to dimes from these two ratios, which is 9:8.

Students who chose (D) may have taken the number of nickels from the first ratio and compared it the number of dimes in the second ratio. Emphasize that since the two ratios do not refer to the same whole, they are not in proportion to each other.

6 0.2, ⅕ Degree of Difficulty: 4

Students can set up a Ratio Chart to solve the problem. Students can then use cross-multiplication to solve for x.

If students answered 28.8, they may have incorrectly set up the Ratio Chart. Students need to make sure they rewrite the ratios in the order in which each ratio is written in the problem.

KAP Wrap

approx. 5 min.

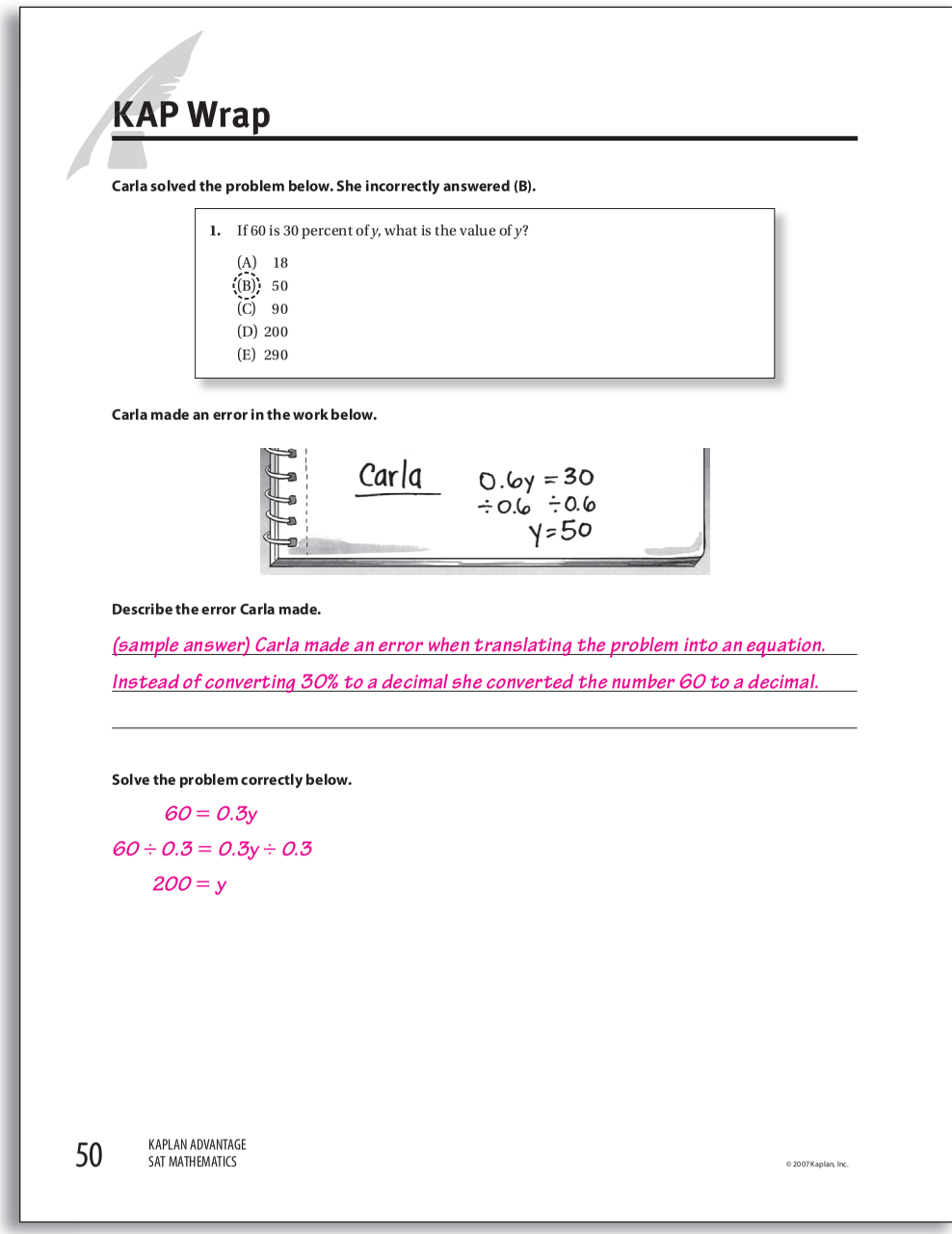
Delivery

Summarize what students have learned about numbers and operations.

- Remind students to use Ratio Charts and percent translations to help solve problems about ratios, proportions, and percents on the SAT.
- To check for understanding, ask students to explain how a percent is related to a ratio. Emphasize that a percent is a special type of part-to-whole ratio, in which the whole is always 100.

Have students work on the KAP Wrap at the end of class or for homework.

- The student work provided has an error that students need to identify. Encourage students to solve the problem themselves before looking for the error in the sample student work.
- Encourage struggling students to refer to the percent translation examples in the lesson to help check the sample student work.



KAP Wrap

Carla solved the problem below. She incorrectly answered (B).

1. If 60 is 30 percent of y, what is the value of y?

(A) 18
(B) 50
(C) 90
(D) 200
(E) 290

Carla made an error in the work below.

Carla

$0.6y = 30$
$\div 0.6 \quad \div 0.6$
$y = 50$

Describe the error Carla made.

(sample answer) Carla made an error when translating the problem into an equation. Instead of converting 30% to a decimal she converted the number 60 to a decimal.

Solve the problem correctly below.

$60 = 0.3y$
$60 \div 0.3 = 0.3y \div 0.3$
$200 = y$

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