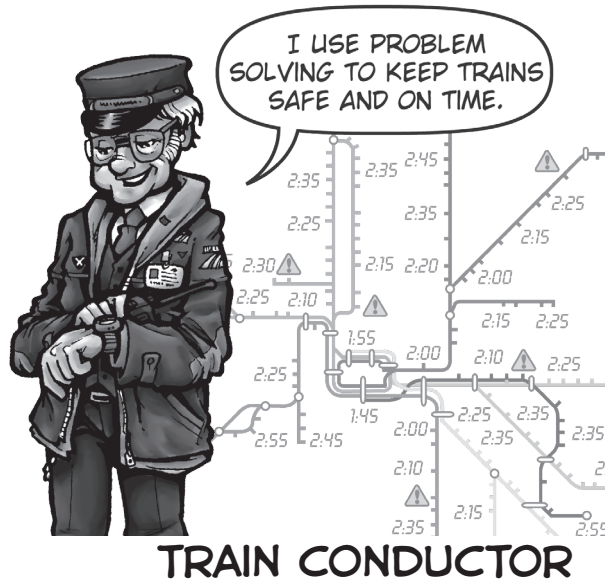


# The 4-Step Method for Problem Solving

## UNIT 1 The 4-Step Method for Problem Solving



**TRAIN CONDUCTOR**

**LESSON 1** Getting Ready . . . . . 15

**LESSON 2** Taking Action. . . . . 25

**LESSON 3** Open-Ended Problems . . . . . 35

### Unit Objectives

**Lesson 1:** Students will

- know that good problem solvers take time to understand math problems before solving them
- be able to understand math problems by underlining important information, restating the question, drawing, and charting

**Lesson 2:** Students will

- know that good problem solvers know many strategies for solving problems and checking their work
- be able to use guiding questions to scaffold the process of solving problems and checking their work

**Lesson 3:** Students will

- know how open-ended problems on the New York State Math Test are scored and how to understand the rubric
- be able to score their own and others' work using a general rubric

# 1 Lesson 1 GETTING READY

## Focus Question

How can you better understand problems before you solve them?

## Thinking KAP

Solve both problems below.

1.  $96 \times 17 = ?$

Answer 1,632

2. All 96 fifth-grade students at an elementary school are taking a field trip to the top of the Empire State Building. The trip will cost \$17.00 per student. What is the total cost of the trip?

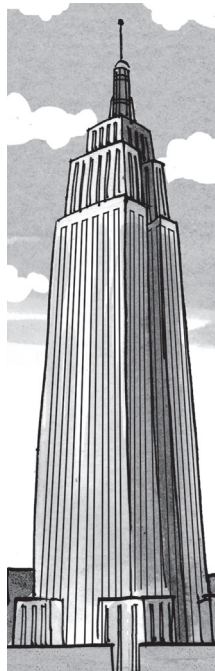
Answer \$ 1,632

What do you notice about problems 1 and 2?

*They have the same answer, and I did the same computation.*

Which problem was more difficult to solve?

*(sample answer) Problem 2, because I had to translate the word problem into math before computing.*



UNIT 1: THE 4-STEP METHOD FOR PROBLEM SOLVING  
LESSON 1: GETTING READY

15

## Thinking KAP

approx. 4 min.

## Delivery

Have students complete the Thinking KAP activity independently.

Alternatively, you may choose to have students complete this activity in pairs.

Lead a discussion about the differences between computation problems and word problems.

Have students share their responses to the activity. The discussion should be guided by student responses, but you may wish to address the following points:

- Even though both problems involved the same computation, problem 2 involved the elements of reading and understanding, which made it more difficult.
- Pure computation problems are usually easier for students to solve, while comprehension problems are often very difficult.
- You can assess students' knowledge of problem solving by asking the following questions:
  - What strategies do you already know for understanding word problems?
  - When you see a word problem, what is the first thing you do?

## Teacher's Note

The first two steps of the 4-Step Method for Problem Solving are aligned to the following process standards:

### STEP 1: Understand the problem.

**Problem Solving:** 5.PS.1, 5.PS.3, 5.PS.9, 5.PS.17

**Communication:** 5.CM.11

### STEP 2: Analyze important information.

**Problem Solving:** 5.PS.2, 5.PS.6, 5.PS.7, 5.PS.8, 5.PS.11, 5.PS.15, 5.PS.18


**Connections:** 5.CN.4, 5.CN.5

**Representation:** 5.R.1, 5.R.3, 5.R.4, 5.R.6

UNIT 1

UNIT 1

# Instruction

 approx. 6 min.

## Delivery

### Introduce the 4-Step Method for Problem Solving.

Read the introductory text with students. Emphasize the following points:

- Many problems on the Math Test will be word problems.
- With word problems, it is important that students take time to understand the problem and analyze the information they are given before trying to solve the problem.

### Guide students through each step of the 4-Step Method for Problem Solving.

Have students read the steps to themselves. Then, ask a volunteer to read the steps aloud. There is no need to teach each step of the method here, as you will go into each step in depth over the course of this unit.

### Compare the 4-Step Method for Problem Solving to other methods students have seen before.

Explain that there are many good methods for problem solving. In fact, students may already have encountered some in their mathematics textbooks. Emphasize that no single method is the best one. Students will learn Kaplan's method in this unit; however, if they are already more comfortable with a slightly different method, they can feel free to use that method when they solve problems in later units.

**ELL**

Focus on the verbs in each step of the 4-Step Method for Problem Solving. Emphasize the difference between *understand* and *analyze*. In Step 1, "understand" simply means knowing what the problem is asking and how the problem is like others students have seen before. In Step 2, "analyze" means working with the information in more depth, so that students can decide how to use the information to solve the problem.



## Instruction

### Getting Started

Welcome to *Kaplan Advantage!* In each lesson of this program, you will learn strategies for solving problems. The best place to start is by learning a method that can help you with *any* problem—the 4-Step Method for Problem Solving.

Many of the questions on the test will be word problems. The 4-Step Method for Problem Solving will help you understand and work through word problems, so you can solve them more easily.

### The 4-Step Method for Problem Solving

- 1** Step 1: Understand the problem.
- 2** Step 2: Analyze important information.
- 3** Step 3: Plan and solve.
- 4** Step 4: Check your work.

How is the 4-Step Method for Problem Solving like methods you have used before? How is it different?

*(Answers will vary.)*

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## Understanding the Problem

Before you solve a math problem, you need to figure out what the problem is basically about and what you are being asked to find.

### 1 Step 1: Understand the problem.

- Scan the problem.
- Restate the question in your own words.

### Scan the Problem

When you see a math problem, the first thing you should do is read it once through quickly. As you read, try to get a general sense of what the problem is about. How is this problem like others you have solved before? How is it different?

### Restate the Question in Your Own Words

To make sure you understand the problem you are being asked to solve, restate the question in your own words.

### TRY IT OUT Complete Step 1 for the problem below.

1 Allie's tomato plant is  $1\frac{1}{4}$  feet tall. If the plant grows

5.M.5 1 foot taller, what will its total height be in inches?

- A
- B
- C
- D

The answer choices have been covered up. Don't solve the problem in Step 1!

What is the problem basically about? finding the height of Allie's

plant

Restate the problem in your own words. How tall is Allie's plant, in

inches, after it grows 1 foot?

## Instruction

approx. 6 min.

## Delivery

### Introduce Step 1 of the 4-Step Method for Problem Solving.

Read the introductory text and the boxed information with students. Emphasize that Step 1 is not about solving the problem. It is simply about getting a basic understanding of the problem.

### Discuss "scanning the problem" in more depth.

Emphasize that in *scanning*, students should quickly read the problem once. Students do not need to understand every part of the problem the first time they read it. After scanning, they should be able to answer the following questions:

- *What is the problem basically about?*  
Make sure students focus on the mathematics in the problem, not the context, or story.
- *How is this problem like others I have seen? How is it different?*  
The more problems students solve, the more easily they will be able to classify problems into useful categories. Having these categories at their fingertips will make students more efficient problem solvers.

### Discuss "restating the question in your own words" in more depth.

Emphasize that the process of restating the question will help students understand what the problem is asking them to find.

### Guide students through the Try It Out exercise.

Read the problem with students, pointing out that the answer choices have been covered up to remind them not to solve the problem. Then, have students complete the Step 1 questions under the problem independently. Allow students to share their responses.

### Teacher's Note

Students might simply identify that the problem is about the measurement of Allie's plant, without mentioning the words *addition* or *conversion*. Do not correct these students. They will develop more useful categories on their own, as their classification schemes increase in sophistication.

However, some students may say that this problem is about gardening. Urge these students to think about the *math* in the problem, not the context, or story.

## Instruction

 approx. 6 min.

### Delivery

#### Introduce Step 2 of the 4-Step Method for Problem Solving.

Read the introductory text and the boxed information with students. Emphasize that Step 2 is also not about solving the problem. Instead, it is a chance for students to pull out the important information and work with it in a way that helps them understand it better.

#### Explain what a “clue” is in more depth.

Read the text below the “What is a clue?” subheading with students. Emphasize that clues are words, numbers, and phrases that convey important *mathematical* information.

#### Guide students through the Try It Out exercise.

Read the problem aloud, and ask students to raise their hands when they hear a clue. Then, discuss what each clue tells students about the problem.

- The first clue, “ $1\frac{1}{4}$  feet,” has been explained on the student page. Have a student read this explanation aloud.
- The second clue, “1 foot taller,” has been filled in for students; however, they must complete the explanation. Have each student write an answer independently. Then, ask students to compare their answers in pairs. Finally, have a few pairs share their explanations with the class.
- For the third clue, students should select a remaining clue in the problem. If time allows, you may wish to make a list of student responses on the board.

Explain that these clues provide all the information needed to solve the problem. However, the second part of Step 2 (rewriting the important information in a helpful way) will help students see how to *use* the clues they have underlined.

#### Teacher’s Note

When students underline numbers in a problem, they should also underline the units. For example, “ $1\frac{1}{4}$ ” by itself does not convey useful mathematical information; “ $1\frac{1}{4}$  feet,” however, conveys information students can use to solve the problem. This will help students recognize when they need to convert units to solve a problem.

### Analyzing the Important Information


In Step 2, you will read the problem again—this time more carefully. As you read, you will underline the clues.

#### **2** Step 2: Analyze important information.

- Underline the clues.
- Rewrite the important information in a helpful way.

#### What is a clue?

A clue tells you about the math in a problem, not the story. Think of a clue as something that could help you solve the problem.

**TRY IT OUT**  **Underline the clues in the problem. Then, explain three of the clues on the lines provided.**

**1** Allie’s tomato plant is  $1\frac{1}{4}$  feet tall. If the plant grows **5.M.5** 1 foot taller, what will its total height be in inches?

- A  $2\frac{1}{4}$  inches
- B 15 inches
- C** 27 inches
- D 28 inches

The clue  $1\frac{1}{4}$  feet tells me the height that Allie’s plant is at first.

The clue 1 foot taller tells me how much taller the plant grows.

The clue total height / in inches tells me I need to add the two heights together / convert feet to inches.

## Rewriting the Important Information

It can often be helpful to rewrite the important information in a different way than how it is given to you. Rewriting the information in a diagram or chart can often help you better understand the problem.

### Draw It

One way to rewrite important information is to draw a simple diagram to help you understand the information you are given. Diagrams are helpful for problems about parts and totals. They are also helpful for many measurement and geometry problems.

### Chart It

Another way to organize information is to make a chart. Charts are most helpful for problems that involve changes over time, patterns and relationships, and data from surveys or experiments.

**TRY IT OUT** Make a diagram or chart to rewrite the important information in the problem on page 18.

(sample diagram)

1 foot	1 foot	$\frac{1}{4}$ foot
--------	--------	--------------------

12 inches	12 inches	3 inches
-----------	-----------	----------

$$12 + 12 + 3 = 27 \text{ inches}$$

(sample chart)

	Now	After Growing
Feet	$1\frac{1}{4}$	$2\frac{1}{4}$
Inches	15	?

## Instruction

approx. 6 min.

## Delivery

**Discuss “rewriting the important information” in more depth.**

Read the introductory text and the text under the two subheadings. Emphasize the following points:

- Rewriting information helps people understand it better. Ask students to recall a time when rewriting information has increased their own understanding of something (e.g., taking notes).
- Drawing and charting are two ways students can rewrite important information in a math problem. Some people prefer to draw; others prefer to chart. For many people, the strategy they use depends on the problem.


**Guide students through the Try It Out exercise.**

Ask half the class to draw a diagram, and ask the other half to make a chart. Then, pair students from each group, and allow them to compare their representations to their partners'. Ask students the following questions:

- How is your partner's representation similar to yours?
- How is your partner's representation different from yours?
- Which representation would you rather use to analyze the important information in this problem? Why?

Close the discussion by emphasizing that neither strategy is inherently better; how students rewrite important information in a word problem is a personal choice for each problem.

# Independent Practice

 approx. 12 min.

## Delivery

**Have students complete the Independent Practice.**

As students work, observe and assist when necessary. Redirect students as needed by asking them questions about their work. Effective questions include the following:

- What is the problem basically about?
- Can you restate the question in your own words?
- What are the clues in the problem? What does each clue tell you?
- Can you make a diagram or a chart to help you analyze the important information?

## Answers

**1 A**

Students should compare 6 bears to 24 stuffed animals and create the ratio  $\frac{6}{24}$ . Students should then simplify the fraction to lowest terms.

*Related Performance Indicator: 5.N.19*



Students who selected (B) or (C) may have compared parts to parts rather than comparing the number of bears to the total number of stuffed animals.

**2 C**

Students should first identify the rule that each shape is rotated  $90^\circ$  counterclockwise from the previous one. Students should rotate the final shape another  $90^\circ$  counterclockwise to find the solution.

*Related Performance Indicator: 5.A.8*



# Independent Practice

Use the 4-Step Method for Problem Solving to solve the problems in this section.

**1** Ming has 24 stuffed animals in her room. Of those, 6 are bears. What fraction represents the ratio of bears to stuffed animals? Simplify the fraction to lowest terms.

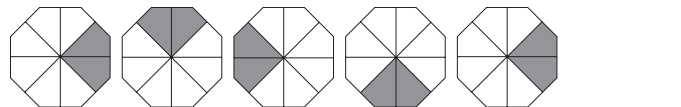
5.N.19

- A**  $\frac{1}{4}$
- B**  $\frac{6}{18}$
- C**  $\frac{2}{5}$
- D**  $\frac{3}{4}$

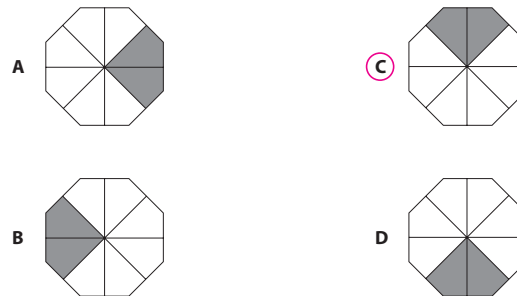
**hint** Remember to simplify both the numerator and denominator.

**2** Farhad made the following pattern.

5.A.8



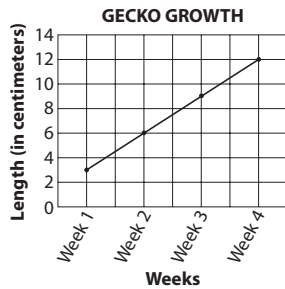
Which shape should go in the blank above?



**hint** Write out the pattern's rule to help you figure out what shape comes next.

- 3 Lee Ann marks the length of her pet gecko as he grows on the line graph below.

5.S.4



According to the graph, how many centimeters has Lee Ann's gecko grown each week?

- A 3 centimeters
- B 6 centimeters
- C 9 centimeters
- D 12 centimeters

**hint** → Pay close attention to the clue "each week."

- 4 Rebecca read the train schedule below for the train she is taking from New York to Rome.

5.M.7

Depart from:	Arrive at:		
New York	Albany	Rome	Buffalo
6:15 A.M.	8:45 A.M.	11:00 A.M.	2:00 P.M.

How long will Rebecca be on the train from New York to Rome?

- A 2 hours 15 minutes
- B 2 hours 30 minutes
- C 4 hours 15 minutes
- D 4 hours 45 minutes

**hint** → You only need some of the information from the table to answer this problem. Circle the relevant information.

## Answers

3 A

Students should look at the shape of the line graph to see how it is changing. Students should recognize that it is increasing by a constant amount each week. Students should then use the graph labels to find that the height is always 3 centimeters more than the week before.

*Related Performance Indicator: 5.S.4*

4 D

Students should first identify the departure time from New York and the arrival time in Rome on the chart. Then, students should count up from 6:15 to 11:00.

*Related Performance Indicator: 5.M.7*



Students who selected (B) may have found the time it took to travel from New York to Albany. Students who selected (A) may have found the time from Albany to Rome. Encourage these students to underline the clues in the problem and circle the appropriate times on the chart to ensure that they are answering the correct question.

## Answers

5 B

Students should find the factors for each number in the question. The factors of 27 are {1, 3, 9, 27}. The factors of 45 are {1, 3, 5, 9, 15, 45}. Students should then compare the factors and select the largest factor that both numbers have in common.

*Related Performance Indicator: 5.N.16*

6 B

Students should recognize that the term *mean* is a clue to find the arithmetic average. Students should first find the sum of all of the numbers in the set to get 425. Students should then divide the sum by the number of terms, 5.

*Related Performance Indicator: 5.S.3*

7 A

Students should recognize that a number written next to a variable indicates that the two are multiplied. They can ask themselves, "What number, when multiplied by 5, equals 60?"

*Related Performance Indicator: 5.A.4*



Students who selected (D) may have multiplied both sides of the equation by 5 instead of dividing. Remind students to reread the problem to see if their answers make sense in the context of the problem.

5 What is the greatest common factor (GCF) of 27 and 45?

5.N.16

- A 3
- B 9
- C 15
- D 1,215

**hint** Draw a factor tree for each number.

6 Victor had the following grades on his last five math tests.

5.S.3

87, 74, 83, 89, 92

What is the mean (average) grade of Victor's math tests?

- A 83
- B 85
- C 87
- D 92

**hint** Underline the clues to be sure that you answer the right question.

7 Mariam made 5 batches of muffins. She made a total of 60 muffins. She used the equation below to determine the number of muffins in each batch,  $b$ .

5.A.4

$$5b = 60$$

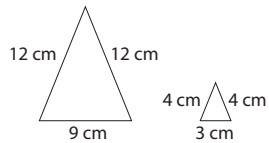
How many muffins were in each batch?

- A 12
- B 55
- C 65
- D 300

**hint** Use what you know about basic number facts to help you solve the problem.

- 8 The diagram below shows two similar isosceles triangles.

5.6.3



Which fraction represents the ratio of the measure of the smaller isosceles triangle to the measure of the larger isosceles triangle?

- A  $\frac{1}{12}$
- B  $\frac{3}{8}$
- C  $\frac{1}{3}$**
- D  $\frac{4}{9}$

**hint** Remember to compare corresponding sides when looking at two similar triangles.

- 9 Jenna ordered baseball caps for her school's team. She made this chart to keep track of the sizes she will need.

5.5.4

BASEBALL CAP SIZES

Size	Number of Baseball Caps
Small	
Medium	
Large	
Extra-Large	

KEY
= 12 caps

Jenna decides to order some Extra-Small caps. She needs more Extra-Small caps than Extra-Large caps, but less than the number of Small caps. How many Extra-Small caps could Jenna order?

- A 3 caps
- B 30 caps
- C 35 caps**
- D 50 caps

**hint** Remember to use the key in a pictograph problem.

## Answers

8 C

Students should compare the corresponding sides of the smaller triangle to the larger triangle as a fraction. Students should then simplify the fraction to lowest terms. Note: students may compare either set of corresponding sides ( $\frac{4}{12}$  or  $\frac{3}{9}$ ). Both simplify to  $\frac{1}{3}$ .

*Related Performance Indicator: 5.G.3*

9 C

Students should first find the number of Extra-Large and Small caps ordered by using the key to read the pictograph. Students should then select the answer choice that is greater than 30 and less than 48.

*Related Performance Indicator: 5.S.4*



Students who selected (A) may not have paid close attention to the key. Remind these students that the key is just as important as the data in a pictograph.

## KAP Wrap

 approx. 5 min.

### Delivery

#### Summarize what students have learned about Steps 1 and 2 of the 4-Step Method for Problem Solving.

Students have learned strategies for understanding problems and analyzing the important information. Remind students that they should always take time to complete these steps before solving any math problem.

Ask the following questions to assess students' understanding of the lesson:

- When you scan a math problem, what should you be looking for?
- Why do you need to restate the question in your own words?
- What is a “clue” in a math problem? What is not a clue?
- What strategies have you learned for rewriting the important information in a helpful way?

#### Have students identify the error in the sample work shown and solve the problem correctly.

If time allows, review the critical errors in the sample student work and what students can learn from these errors. You may wish to emphasize the following points:

- Students should take the time to reword the problem. Making sure that students know what “perimeter” means will make solving the problem easier. Students may not be familiar with the perimeter formula, but with an understanding of perimeter they can still solve this problem.
- The error Kameron made (calculating the length of only two sides) is a common one. Test writers know this, so they always include an answer choice like (B).

## KAP Wrap

Kameron solved the problem below. He incorrectly answered (B).

- 1** Cody is making a picture frame for his favorite picture. The dimensions of his picture are shown below.

5.A.6



What is the total length, in inches, of the materials he will need to make the frame?

$$\text{perimeter of rectangle} = (2 \times \text{length}) + (2 \times \text{width})$$

- A 18 inches
- B 42 inches
- C 64 inches
- D 84 inches

Describe the error Kameron made. Kameron forgot to multiply the length and width by two before adding. He only found the length of two sides of the rectangle, not all four. He needs to multiply his answer by two to get 84 inches.

What is the correct answer? (D)

## Extension Activities

### Reteaching

Use a picture to teach the strategies for understanding the problem and analyzing the important information. Show students an illustration or photograph that depicts a problem situation (you may wish to use a Web site such as <http://images.google.com> to find an appropriate picture). Have students practice scanning the picture, asking themselves the questions “What is this picture basically about?” and “How is the problem in this picture like others I have seen? How is it different?” Make sure that students read the problem out loud. Have students circle or underline the elements of the image that provide them with specific clues as to what is happening in the situation. Finally, have students make a diagram, chart, or list to organize the clues in the picture. Point out that students have a much deeper understanding of the picture than they did when they first saw it. Explain that students can use this same process to analyze word problems and understand them in more depth.

### Journal Prompt

Have students respond to the following prompt:

What do you think is most difficult about facing a word problem on a math test? How can understanding the problem and analyzing the important information make these problems easier for you?

### Challenge Problem

- 1** Miranda is baking cookies for her best friend’s birthday party. She needs to make 75 cookies, but the recipe below only makes 25.

5.N.21

#### COOKIE RECIPE

Flour	2 cups
Granulated Sugar	$1\frac{1}{4}$ cups
Brown Sugar	$\frac{3}{4}$ cup
Butter	$\frac{1}{2}$ cup
Eggs	2
Milk	$\frac{1}{3}$ cup
Chocolate Chips	12 ounces

To make all 75 cookies, how many more cups of granulated sugar will Miranda need than cups of brown sugar?

**Show your work.**

**Answer** \_\_\_\_\_ cups

**Answer:**  $1\frac{1}{2}$