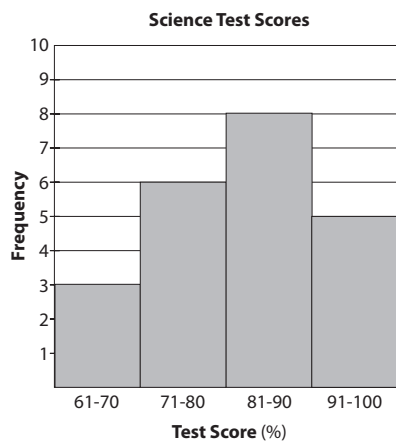


3 Lesson 3 HISTOGRAMS

Thinking KAP

This histogram shows the test scores of students on a recent science test.



1. How many students received between 71% and 80% on the test?

6

2. How many students received a grade higher than 80% on the test?

13

3. Why is this data shown using a histogram instead of another type of graph?

A histogram is used to show frequencies.

Thinking KAP

approx. 4 min.

Delivery

Have students complete the Thinking KAP activity independently.

Alternatively, you may choose to have students complete this activity in pairs.

Lead a discussion about the data in the histogram.

Have students share their responses to the activity. The discussion should be guided by student responses, but you may wish to address the following points:

- Students should try to recognize the data in the histogram to answer the questions. Note those students who do not know how to read the histogram. Students will create histograms on the next page, so do not spend too much time reviewing them here.
- You can assess students' knowledge of histograms by asking students the following questions:
 - What does the horizontal scale tell you?
 - What interval is used to display the data?
 - What does the vertical scale tell you?
 - What does each bar show?
- Point out that unlike a bar graph, there are no spaces between the bars on a histogram.

Activating Prior Knowledge

This Thinking KAP activity reviews the following middle-school standards:

- 6.S.4** Determine and justify the most appropriate graph to display a given set of data (pictograph, bar graph, line graph, histogram, or circle graph)
- 7.S.6** Read and interpret data represented graphically (pictograph, bar graph, histogram, line graph, double line/bar graphs or circle graph)

Instruction

approx. 6 min.

Delivery

A.S.5 Construct a histogram, cumulative frequency histogram, and a box-and-whisker plot, given a set of data

Discuss how to create a histogram from data.

Read the introductory text with students.

- Explain that a histogram uses bars to show data organized in equal intervals. Histograms show continuous data so there are no spaces between the bars.

Guide students through the example problem.

As the first step to creating a histogram, students should complete the frequency table. Students are given the intervals to be used to complete the interval column. Have them count the numbers that belong in each interval to complete the frequency column.

- Ask students, “Which values in the data belong in the interval 1–4?” Have students count the values and write the number in the “Frequency” column.
- Highlight that the intervals are equal sized—each interval includes four numbers. The first and last numbers in each interval, as well as the numbers between the two, are included in that interval.
- Explain that if students have to determine their own intervals on the exam, students should look at the range and determine the best even-sized intervals to display the data in about five intervals.

Guide students through the Try It Out exercise.

Students use the frequency table they created in the example to construct a histogram. Make sure students label and title their graphs.

- Explain that data in a histogram gives a better visual display than data in a frequency table. This makes it easier to interpret data in a histogram.
- Identify struggling students and redirect them with the following questions:
 - What numbers should you use to label the horizontal axis? the vertical axis?
 - How high should the first bar reach?



Instruction

Inside the Test!

You will more likely be asked to construct a histogram in an open-ended question rather than to analyze a given histogram in a multiple-choice question.

Histograms

In order to create a histogram from data, you need to draw a frequency distribution table. Then you can use the table to draw the histogram.

Example

- 1 The following data represents the number of days absent last year for each of 20 students.

5, 6, 2, 14, 8, 4, 4, 7, 5, 11, 10, 2, 1, 9, 13, 4, 12, 17, 1, 6

Create a frequency distribution table. Use intervals of 1–4, 5–8, 9–12, 13–16, and 17–20.

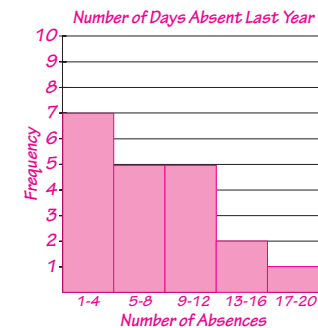
The intervals include the endpoints.

Interval	Frequency
1-4	7
5-8	6
9-12	4
13-16	2
17-20	1

TRY IT OUT

Solve the problem below.

- 2 Using the frequency distribution table above, construct a histogram on the grid below.



Write “Number of Absences” along the horizontal axis. Write “Frequency” along the vertical axis. Draw a bar to reflect the data in each row of the table. Make sure you title your graph.

REMEMBER

- Unlike a bar graph, there is no space between the bars of a histogram.

Cumulative Frequency Histograms

A **cumulative frequency histogram** shows the running total of frequencies. Each interval includes every interval up to that point.

Example

- 3 The following data represents the numbers of days absent last year for each of 20 students.

5, 6, 2, 14, 8, 4, 4, 7, 5, 11, 10, 2, 1, 9, 13, 4, 12, 17, 1, 6

Create a cumulative frequency distribution table. Use intervals of 1-4, 1-8, 1-12, 1-16, and 1-20.

The cumulative frequency column should be the total number of terms in the data set.

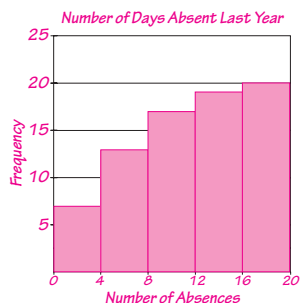
Each interval includes the other intervals before it.

Interval	Frequency	Cumulative Frequency
1-4	7	7
1-8	6	13
1-12	4	17
1-16	2	19
1-20	1	20

TRY IT OUT

Solve the problem below.

- 4 Using your table above, construct a cumulative frequency histogram on the grid below.



Inside the Test!

You will need to label each axis and title your graph to ensure you receive full credit on the Regents Exam.

Instruction

approx. 6 min.

Delivery

Explain that students may see cumulative frequency histograms on the test.
Read the introductory text with students.

- Point out the difference between a cumulative frequency histogram and the histogram that students constructed on the previous page. In a cumulative frequency histogram, the frequency in each interval includes the frequencies in the previous intervals.

Teacher's Note

Students should note the terms *cumulative* meaning collective, and *frequency* meaning occurrence to help them remember frequency tables/histograms and cumulative frequency tables/histograms.

Guide students through the example problem.

Students should use the same data in the example on the previous page to complete a cumulative frequency table.

- Students should note that *cumulative* means collective. This will help them distinguish frequency tables/histograms from cumulative frequency tables/histograms.
- Point out that instead of counting the pieces of data in each interval again to complete the cumulative frequency column, students can add the frequencies in the frequency column (for that frequency and the previous frequencies).
- Use the second callout as a way to show students how they can check to make sure they have the correct cumulative frequencies.

Guide students through the Try It Out exercise.

Students should use their cumulative frequency table to construct a cumulative frequency histogram.

- Identify struggling students and redirect them with the following questions:
 - What numbers should you use to label the horizontal axis? the vertical axis?
 - How will the bars in a cumulative frequency histogram differ from that of a frequency histogram?

REMEMBER

- In a cumulative frequency histogram, each bar will be the same height or higher than the previous bar.

Instruction

approx. 6 min.

Delivery

A.S.9 Analyze and interpret a frequency distribution table or histogram, a cumulative frequency distribution table or histogram, or a box-and-whisker plot

Discuss analyzing and interpreting data in frequency histograms and cumulative frequency histograms. Read the introductory text with students.

- Explain that students may be asked to analyze a frequency histogram or cumulative frequency histogram on the Regents Exam. Students will need to understand how the data is represented on the histogram to answer the questions.

Guide students through the example problem.

Work through the question with students.

- Point out that the table shows cumulative frequencies, so each interval will include values contained in previous intervals.
- Explain that the table shows the frequencies for the runners taking less time at the bottom of the table.
- Use the callouts to help students understand which intervals they need to use to answer the problem.

! Students may try to subtract the frequencies in the intervals 20–40 and 20–50. Ensure they understand that it is the frequency for 20–30 that they should not include in the interval for 20–50.

Guide students through the Try It Out exercise.

Read through the question with students. Remind students that the bars show cumulative frequencies.

- Identify struggling students and redirect them with the following questions:
 - Which interval(s) do you need?
 - Can you immediately eliminate any answer choices?
 - How should you use the cumulative frequencies you found?

Inside the Test!

If you are not asked for the cumulative total, you can eliminate the answer choice that includes the total height of the last bar in your interval.

Analyzing Histograms

You can use frequency histograms and cumulative frequency histograms to analyze and interpret data.

Example

- 5 The cumulative frequency table below shows the times, in seconds, of runners in a race.

Interval (time in seconds)	Cumulative Frequency (Number)
20–30	7
20–40	15
20–50	27
20–60	41
20–70	49

How many runners had a time greater than 30 seconds but less than 51 seconds?

- (1) 27 (3) 15
(2) 20 (4) 12

This limit eliminates times in the first interval, 20–30.

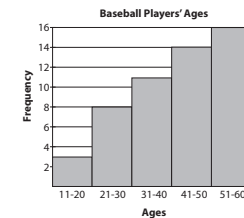
This limit indicates the times in question are in the interval 31–50.

Subtract the first interval from the total in the greatest interval included.

$$27 - 7 = 20$$

TRY IT OUT Solve the problem below.

- 6 This cumulative frequency histogram shows the ages of the players on a city baseball team.



How many players are at least 31 years old but less than 40 years old?

- (1) 3 (3) 11
(2) 6 (4) 25

REMEMBER

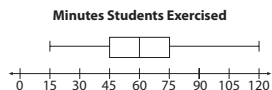
- To interpret a frequency histogram, you can simply read the height of the correct interval. To interpret a cumulative frequency histogram, you may need to subtract to find the size of the correct interval.

Analyzing Box-and-Whisker Plots

A box-and-whisker plot divides data into quartiles. These plots can be useful in interpreting the distribution of data. The more closely the data groups together, the closer the data points are in that interval.

Example

- 7 This box-and-whisker plot below shows the number of minutes students spent exercising last week.



What are the minimum and maximum numbers of minutes students exercised?

15, 120

What is the number of minutes students exercised at the 25th percentile? 45

What is the number of minutes students exercised at the 50th percentile? 60

What is the number of minutes students exercised at the 75th percentile? 75

The 25th percentile is the same as the first quartile.

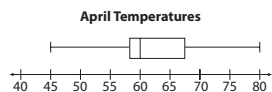
The 50th percentile is the same as the median of the data.

The 75th percentile is the same as the third quartile.

TRY IT OUT

Solve the problem below.

- 8 The box-and-whisker plot below shows the temperatures during the month of April.



In which range are the data points the most concentrated?

- (1) between the minimum and the first quartile
- (2) between the first quartile and the second quartile
- (3) between the second quartile and the third quartile
- (4) between the third quartile and the maximum

Inside the Test!

If asked to find the median when given a box-and-whisker plot, choose the number below the line in the middle of the box.

REMEMBER

A quartile is a number, not a range of values. For example, a value can be above or below the third quartile, but not in the third quartile.

Instruction

approx. 6 min.

Delivery

Discuss analyzing and interpreting box-and-whisker plots.

Read the introductory text with students.

- Remind students that a box-and-whisker plot uses medians to divide data into quartiles. The first quartile is the median of the lower half of the data, the second quartile is the median of the entire data set, and the third quartile is the median of the upper half of the data.
- Explain that short boxes or whiskers indicate a clustering of the data, while longer boxes or whiskers indicate that the data is spread out.

Guide students through the example problem.

Work through the question with students.


- Use the callouts to remind students of the quartile representing each percentile. The percentile indicates the percent of values that are above or below that value. For the 25th percentile, for example, 25% of the data are less than the first quartile and 75% of the data are greater.
- Remind students that the whiskers show the minimum and maximum of the data.
- Remind students that since the quartiles are medians of parts of the full set of data, the value at each quartile is a number which may not be a part of the data set.

Guide students through the Try It Out exercise.

Read through the problem with students. Have students work independently or in small groups.

- Identify struggling students and redirect them using the following questions:
 - When the data around the minimum of the plot are close together, what happens to the whisker?
 - When the data in the lower half of the plot are close together, what happens to the box?

Independent Practice

 approx. 12 min.

Delivery

Have students complete the Independent Practice.


As students work, observe and assist when necessary. Redirect students as needed by asking them questions about their work. Effective questions include the following:

- Does this display show frequency or cumulative frequency?
- Which intervals should you use?
- Can you immediately eliminate any answer choices?

1 (2)

Students should recognize that this is a frequency chart, and they must add all of the frequencies for the total.

Related Performance Indicator: A.S.9

 Students may use the highest interval instead of the frequency. Point out that each interval shows an age range, and the frequency shows the number of volunteers in that range.

2 (3)

Students should read the cumulative frequency for the interval 1–30 in the chart.

Related Performance Indicator: A.S.9



Independent Practice

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, select the numeral preceding the word or expression that best completes the statement or answers the question.

1 The accompanying table shows the age groups for members of a volunteer center.

Use this space for computations.

Interval	Frequency
10–19	14
20–29	12
30–39	4
40–49	8
50–59	5

What is the number of volunteers represented?

- (1) 14 (3) 49
(2) 43 (4) 59

hint  The intervals represent the age groups of the members.

2 The accompanying table shows the amount of money donated to a homeless shelter by 100 high-school students.

Interval (dollars)	Frequency	Cumulative Frequency
1–10	14	14
1–20	12	26
1–30	4	30
1–40	8	38
1–50	5	43

How many students donated \$30 or less?

- (1) 4 (3) 30
(2) 16 (4) 70

hint  The last cumulative frequency value is the total number of students who donated.

Answers

3 (1)

Students should subtract the cumulative frequency in the interval 25–35 from the cumulative frequency in the interval 25–45.

Related Performance Indicator: A.S.9

4 (3)

Students should know that the second quartile in a box-and-whisker plot is the median of the data in the plot.

Related Performance Indicator: A.S.9

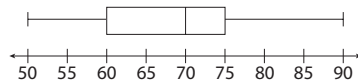
3 The accompanying table shows the amount of time, in minutes, in which cyclists completed a marathon.

Interval (minutes)	Frequency	Cumulative Frequency
25–30	2	2
25–35	4	6
25–40	9	15
25–45	5	20
25–50	1	21

How many cyclists took longer than 35 minutes but less than 46 minutes to complete the marathon?

- (1) 14 (3) 35
(2) 18 (4) 41

4 The accompanying box-and-whisker plot represents the scores earned on a history test.



What is the median score?

- (1) 60 (3) 70
(2) 65 (4) 75

Answers

For questions 5 and 6, use the specific criteria to award credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

5 See student page for answer.

[2] Students have titled both axes, used a correct scale for both axes, and used bars that represent the frequency of each interval.

[1] Appropriate work is shown, but one error is made, such as missing labels.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Related Performance Indicator: A.S.5

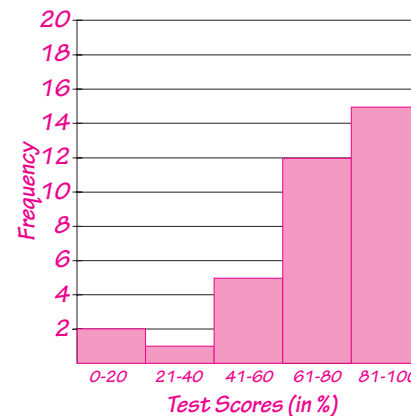
Answer all questions in this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The credit value is listed in brackets after each question. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

5 The accompanying table shows the test scores, in percentages, for the students in a statistics class.

Interval	Frequency
0–20	2
21–40	1
41–60	5
61–80	12
81–100	15

Using the frequency distribution table above, construct a histogram on the grid below. [2]

hint Write the frequency along the vertical axis.



Answers

6 See student page for answer.

[2] 1, 6, 13, 19, 21, respectively in table; and appropriate work is shown, such as adding to find cumulative frequency.

[1] Appropriate work is shown, but one conceptual error is made, such as adding only the frequency in the previous interval, and not all previous intervals.

or

[1] Appropriate work is shown, but one computational error is made, such as an error in addition.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Related Performance Indicator: A.S.9

6 The accompanying table shows the heights, in inches, for the students on a junior-high softball team.

Interval (inches)	Frequency	Cumulative Frequency
55–57	1	1
58–60	5	6
61–63	7	13
64–66	6	19
67–69	2	21

Using the data, complete the cumulative frequency column above. [2]

KAP Wrap

approx. 5 min.

Delivery

Summarize what students have learned about constructing and analyzing histograms, and analyzing box-and-whisker plots.

Students should know that understanding how histograms and box-and-whisker plots are constructed can help them analyze these graphs.

- Ask students the following questions to assess their understanding of the lesson:
 - What kind of visual displays are you responsible for constructing and analyzing on the Regents Exam?
 - What tables or graphs did you find difficult to construct?
 - What tables or graphs did you find difficult to analyze?
- Students should make examples of the tables and graphs they have learned in this unit in their notebook.
- You may want them to write a problem that requires constructing a frequency histogram and a cumulative frequency histogram. Students can exchange their problems with partners and create the tables and histograms.

Have students work on the KAP Wrap at the end of class or for homework.

Students should focus on the communication of their answers throughout this KAP Wrap. Have students try to make fully complete answers, showing all their work. Students should keep in mind that more complete solutions on the Regents Exam will give them the highest number of credits.

Students should then complete the Reflect at the bottom of the page.

Alignment to New York State Core Curriculum Process Standards

This KAP Wrap activity should focus on the following process standards:

- A.PS.4** Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)
- A.PS.8** Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

KAP Wrap

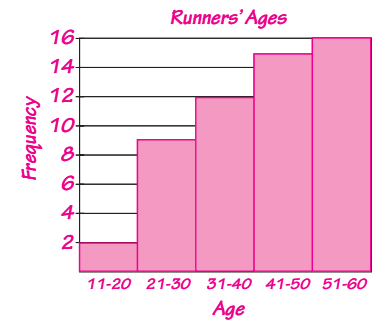
Challenge Problem

This list shows the ages of participants in a recent 5-kilometer run.

29, 26, 35, 18, 40, 43, 21, 24, 45, 28, 25, 32, 33, 54, 19, 22

Choose intervals and fill in the cumulative frequency table below. Then use the table to draw a cumulative frequency histogram.

Interval	Frequency	Cumulative Frequency
11-20	2	2
21-30	7	9
31-40	4	13
41-50	2	15
51-60	1	16



Reflect

When would you use a histogram? When would you use a box-and-whisker plot?

(sample answer) I would use either graph to represent data that can be measured on an interval scale. A histogram will tell me which interval occurs the most; a box-and-whisker plot will break the data into quartiles.