

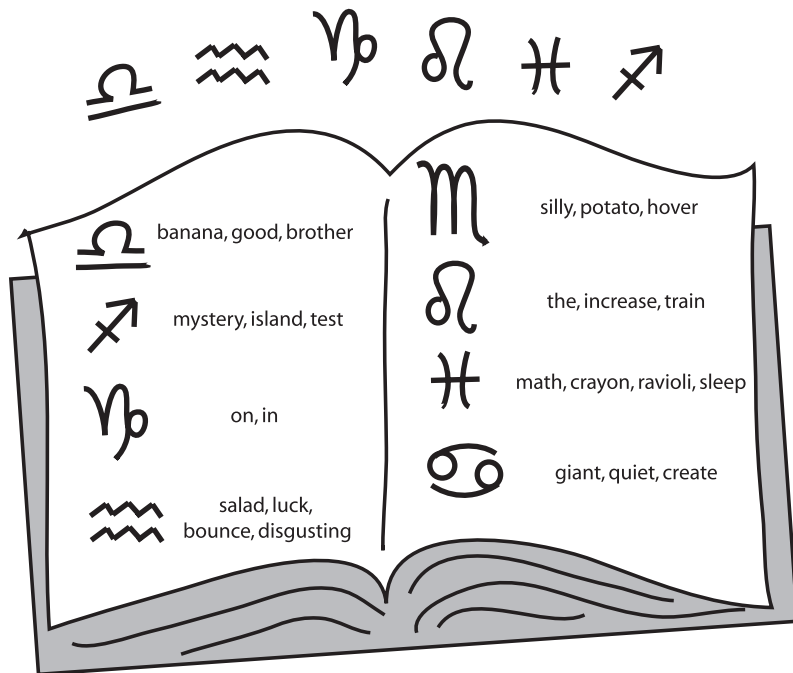
Lesson **A** *Translating Word Problems into Algebra*

Thinking KAP



Your character, Oakthorn, finds a coded message in ancient runes, carved into a giant tree in the center of a forest. A good wizard has given you a book, with which you can decode the message.

Use the code book to translate the message below.



A

Lesson A: Translating Word Problems into Algebra

Task

Students will use a real-world situation to initiate a discussion about translating words into algebraic expressions.

Delivery

Have students complete the Thinking KAP activity independently.

Alternatively, you may choose to have students complete this activity in pairs.

Lead a discussion about translating symbols into words.

Have students share their responses to the activity. The discussion should be guided by student responses, but you may wish to address the following points.

- There are several different ways to translate the sentences; however, only one way makes sense. Similarly, an operation symbol—such as the multiplication sign—can represent more than one word or phrase—such as *multiplied by*, *times*, or *of*, depending on the context in which it is used.
- A symbolic code is an easy way to represent a large amount of information with only a few marks. Similarly, algebraic notation is an efficient way for mathematicians to communicate effectively.

Moving On

“You just decoded information by using a key. In math, symbols are used to represent words and phrases that indicate relationships between numbers.”

Strategy Instruction



Clues for Translation

Algebra is a language, just as English and Spanish are languages. Sometimes, before you solve a word problem, you need to translate that problem into an algebraic expression or equation. Clue words can help you do that. The chart below reviews useful clue words.

Keep in mind

Mathematical terms can also help you translate a problem. For example, the clue word *sum* is a signal to add and the clue word *product* is a signal to multiply.

Clue Word	Symbol	In Words ...	In Algebraic Language ...
is/was/will be	=	The cost <i>is</i> \$10.	$c = 10$
total	+	The <i>total</i> number of boys and girls was 14.	$b + g = 14$
plus	+	An amusement park charges \$10 at the gate and \$2 per ticket.	$10 + 2x$
more than	+	Maggie has \$5 <i>more than</i> Kim.	$m = 5 + k$
less than	-	The height is 2 cm <i>less than</i> the length.	$h = l - 2$
of	×	75% <i>of</i> the regular cost	$0.75c$
per	×	\$0.45 <i>per</i> pound	$0.45p$
twice	×	<i>twice</i> as tall as Ricky	$2r$
times	×	3 <i>times</i> as old as Darius	$3d$
half	÷ or /	<i>half</i> as many pumpkins	$\frac{1}{2}p$ or $\frac{p}{2}$

A

Task

Students will review clue words that help them translate English words into symbolic notation.

Delivery

Help students make the connection between the Thinking KAP activity and translating words into algebraic notation.

Read the introductory text with students. Emphasize that algebra is simply a language for expressing relationships. Students should be familiar with the most common English words and phrases used to express relationships, so that they can translate them into mathematical symbols.

Introduce the list of clue words.

As you list clue words for students, have them guess what symbols could be used to replace them. Guide students in selecting symbols. Emphasize the table of clue words after the initial guessing. Point out any examples that were not covered.

Apply clue words to word problems.

Direct students' attention to the third column of the table. Read examples in this column with students before having them translate similar examples. Try reading entire word problems and have students point out clue words. Guide them as they translate into symbols.

Moving On

“We have practiced finding clue words and changing them into symbols. Now let's match logical groups of words to mathematical expressions.”

Use Logical Groups

When translating from words into algebra, translate in logical groups. Look for clue words to help to identify groups that can be translated into a word or symbol.

For each expression in words, find the mathematical expression that matches. Use clue words to help you. Let p stand for the number of pens Alice has.

- | | | | |
|--------|--|----|---------------|
| ___ 1. | half as many pens as Alice | A. | $2p$ |
| ___ 2. | twice as many pens as Alice | B. | $\frac{p}{2}$ |
| ___ 3. | 7 more pens than Alice | C. | $3p - 7$ |
| ___ 4. | 7 less than 3 times the number of pens Alice has | D. | $7 + p$ |

Try It Out!

Write your own expression for this problem. Then use that expression to solve the problem.

5. The cost of renting a car is \$45 plus \$0.10 for each mile traveled. How much would it cost to rent a car and drive it 50 miles?
- Underline the clue words.
 - Write an expression for the cost of renting any car. Let m stand for the number of miles traveled.
 - Substitute 50 for m in the expression. Find the answer.

keep in mind

Clue words are just guides to help you translate. Take the time to consider whether the algebraic expression or equation makes sense for the problem.

A

Task

Students will translate words into symbolic notation, working in logical groups.

Delivery

Explain how to translate in logical groups.

Read the text with students. Explain that most logical groups contain two parts. Sometimes, a logical group may contain a variable and a number. Other times, a logical group may contain an operation and either a variable or a number.

Use the examples provided to demonstrate logical groups.

Complete the examples together as a class. Read the first example aloud: “half as many pens as Alice.” This example comprises a logical group and is represented by (B). Read the second example with students: “twice as many pens as Alice.” Allow students to identify the matching expression: (A). Have students complete the third and fourth examples in pairs. Point out that the fourth example includes two logical groups: “7 less than” and “3 times the number of pens Alice has.”

1 B

2 A

3 D

4 C

Try It Out: Answers

- 5 The cost of renting a car is \$45 plus \$0.10 for each mile traveled. How much would it cost to rent a car and drive it 50 miles?

$$45 + 0.10m$$

$$45 + 0.10(50) = 45 + 5 = 50$$

Moving On

“Translating can help you identify the correct expression to model an algebraic situation. However, another strategy you can use is Picking Numbers.”

Picking Numbers

For test questions that ask you to identify the expression or equation for a particular problem, you can also use Picking Numbers.

Pick numbers to stand for the unknown values and use those numbers to solve the problem. Then substitute those same numbers into the answer choices to see which answer choices give the same result.

Keep in mind

Whenever a problem has variables in the answer choices, think about whether or not Picking Numbers can help you.

Picking Numbers

- Pick numbers to stand for unknown numbers or variables.
- Compute the answer using the number you picked.
- Substitute the value you picked into the expression in each answer choice. The expression that yields the same value as the one you found is the correct choice.

Try It Out!

Use Picking Numbers to solve the problem below.

1. A bus holds 30 people, and a van holds 10 people. Which of the following expressions shows the number of people that could fill b buses and v vans?
- A. $b + v$
- B. $30b + 10v$
- C. $\frac{bv}{40}$
- D. $40bv$
- E. $10b + 30v$

A

Task

Students will use Picking Numbers to select the correct expression or equation.

Delivery

Introduce Picking Numbers.

Guide students through the elements of the strategy. Emphasize that students can pick any numbers, as long as they use them consistently. Therefore, advise students to pick easy numbers, such as 2 or 3. Discourage students from picking 0 or 1, since these numbers have special identity properties that can make simplifying expressions difficult.

Guide students through the Try It Out exercise, using Picking Numbers to solve the problem.

Read the problem with students. Pick the following numbers:

$$b = 2$$

$$v = 3$$

Calculate the number of people, based on these numbers.

$$\text{Number of people in 2 buses} = 2 \times 30 = 60$$

$$\text{Number of people in 3 vans} = 3 \times 10 = 30$$

$$30 + 60 = 90$$

Then substitute 2 and 3 into each answer choice until you identify the one that yields 90.

(A) $2 + 3 = 5$

(B) $30(2) + 10(3) = 90$ This answer choice is correct.

Try It Out: Answers

1 B

Moving On

“Picking Numbers is a useful strategy when there are expressions in the answer choices. When there are expressions in the problem and numbers in the answer choices, Substitute and Compute can help you.”

Substitute and Compute

Sometimes you do not need to pick numbers to solve problems. Sometimes, a problem will tell you which numbers to pick. When a problem tells you the value of a variable and asks you to find its value, just Substitute and Compute.

Substitute and Compute

- Substitute a value for each variable.
- Use order of operations to simplify.

Here is a review of the correct order of operations.

1. First perform operations in *parentheses*.
2. Next, evaluate *exponents*.
3. *Multiply* and *divide* in order from left to right.
4. *Add* and *subtract* in order from left to right.

The memory device, *PEMDAS*, can help you recall the correct order of operations: *Parentheses, Exponents, Multiply and Divide, Add and Subtract*.

Try It Out!

Substitute and Compute to solve the problem below.

2. If $x = 4$, then what is the value of $6x^2 - 3x^2 + 2x \cdot 5$?

Substitute 4 for x in the expression: $6x^2 - 3x^2 + 2x \cdot 5 =$ _____

Use the order of operations to simplify.

keep in mind

Substitute and Compute can also help you solve function problems. For example, if $f(x) = 2x - 1$, then $f(3)$ can be found by substituting 3 for x in the expression $2x - 1$.

A

Task

Students will simplify expressions by using Substitute and Compute.

Delivery

Introduce Substitute and Compute.

Guide students through the elements of the strategy. Emphasize that this strategy has two parts. First, students should rewrite the expression, substituting the number given for the variable. Then students should simplify. Discourage students from trying to complete both steps simultaneously; this can lead to errors.

Review the correct order of operations.

Read this text with students. Introduce the memory device PEMDAS. Emphasize that multiplication and division are on the same step, or level, and are to be done left to right. Students should not complete all the multiplication first, then all the division. The same is true for addition and subtraction.

Guide students through the Try It Out exercise, using Substitute and Compute to solve the problem.

Together as a class, write the expression with the number 4 substituted for the variable x .

$$\begin{aligned} &6(4)^2 - 3(4)^2 + 2(4) \cdot 5 \\ &= 6(16) - 3(16) + 8 \cdot 5 \\ &= 96 - 48 + 40 \\ &= 88 \end{aligned}$$

Try It Out: Answers

2 88

Moving On

“Now let’s practice together.”

Guided Practice



The 4-Step Method for Problem Solving



STEP 1: Understand the problem.



STEP 2: Analyze important information.



STEP 3: Plan and solve.



STEP 4: Check your work.

Picking Numbers

- Pick numbers to stand for unknown numbers or variables.
- Compute the answer using the number you picked.
- Substitute the value you picked into the expression in each answer choice. The expression that yields the same value as the one you found is the correct choice.

Substitute and Compute

- Substitute a value for each variable.
- Use order of operations to simplify.

A

Guided Practice

Briefly review the content and strategies from this lesson.

Ask a student to summarize what the class has learned. You may wish to use the board or chart paper to record these ideas, particularly the names of new strategies.

There are several recommended ways to present the Guided Practice problems, as listed below. However you choose to deliver this section, be sure to emphasize the systematic and strategic thinking that will help students succeed on Test Day.

1. Model expert test-taking strategies by giving students a window into your thought process. Think aloud as you work through the problems.
2. Allow students to guide you through the systematic approach to the test-taking strategies they have learned. You can serve as a scribe, recording the ideas of the class and facilitating their thought processes.
3. Balance teacher and student interaction by modeling how to approach the first problem, then inviting students to take a more active role in approaching the next one.

Teacher's Note

In this portion of the lesson, it is critical that you model and reinforce systematic and strategic thinking. Be sure to follow the 4-Step Method for Problem Solving for each problem. Also, be sure to model the strategies suggested by the reference text. Although there is almost always more than one way to solve a math problem, these problems were designed as vehicles for specific strategies in this lesson.

1. A salesperson earns $7h + 0.04s$ dollars, where h is the number of hours worked and s is the total amount of her sales. What does she earn for working 15 hours with a total of \$120.50 in sales?
- A. \$109.82
 - B. \$153.20
 - C. \$226.10
 - D. \$231.50
 - E. \$848.32

A

Guided Practice

1 A



STEP 1: Understand the problem.

Scan: This problem is basically about an algebraic expression.

Restate: This question can be restated as, “What will you get if you substitute numbers for h and s and compute?”



STEP 2: Analyze important information.

$$h = 15$$

$$s = 120.50$$



STEP 3: Plan and solve.

Students can use Substitute and Compute to solve the problem.

$$7(15) + 0.04(120.50) = 105 + 4.82 = 109.82$$



STEP 4: Check your work.

Model the check question, “Is your answer reasonable?” Students can use estimation to check their work. The expression has two components. 7×15 is midway between 70 and 140, or about 100. The small number, 0.04, when multiplied by 120.50, will yield a small number. Therefore, it is reasonable that the answer is a bit more than 100.

2. At a grocery store, item A costs \$5 less than item B , and item B costs three times as much as item C . If n is the cost of item C in dollars, what is the cost of item A in dollars?

F. $n - 3$

G. $n - 5$

H. $3n - 5$

J. $3n - 15$

K. $5n$

A

Guided Practice

2 H



STEP 1: Understand the problem.

Scan: This problem is basically about modeling a real-world situation with an algebraic expression.

Restate: This question can be restated as, “Find the cost of A in terms of n .”



STEP 2: Analyze important information.

Item A	$B - 5$
Item B	$3C$
Item C	n



STEP 3: Plan and solve.

Students can use Picking Numbers to solve the problem. For example, suppose that Item C costs \$3. Then, Item B costs three times as much, or \$9. Item A costs \$5 less than B, or \$4.

Students can substitute 3 for n in the answer choices, to find the expression that yields 4.

(F) $3 - 3 \neq 4$

(G) $3 - 5 \neq 4$

(H) $3(3) - 5 = 4$



STEP 4: Check your work.

Model the check question, “Can you solve the problem another way?” Students can prove their answer is correct by trying the remaining answer choices and Eliminating.

(J) $3(3) - 15 \neq 4$

(K) $5(3) \neq 4$

Shared Practice



Use the 4-Step Method for Problem Solving and the strategies you learned in this lesson to solve the problems in this section.

1. If $n = 7$, then $2n + 3n + 4n + 5n + 6n$ is equivalent to:

- A. 70
- B. 84
- C. 98
- D. 126
- E. 140

hint → *Substitute and Compute. Be sure to follow the correct order of operations.*

2. Joan has q quarters, d dimes, n nickels, and no other coins in her purse. Which of the following represents the total number of coins in Joan's purse?

- F. $q + d + n$
- G. $5q + 2d + n$
- H. $0.25q + 0.10d + 0.05n$
- J. $(25 + 10 + 5)(q + d + n)$
- K. $25q + 10d + 5n$

hint → *Pay careful attention to what the question is asking. It is asking for the total number of coins, not the total value of those coins.*

A

Shared Practice

Have students solve the problems in pairs or small groups.

Give students approximately 8 minutes to complete the Shared Practice. Then review.

As students work, observe and assist when necessary.

Circulate and assess students' progress. Provide support for students who may be having difficulty.

Shared Practice: Answers

1 E

Students need to Substitute and Compute, using 7 for n . They need to perform all multiplication before addition, following the order of operations. An optional approach is to add all like terms before substituting, yielding $20n$ as the expression. This option makes the arithmetic easier to do mentally.

$$2(7) + 3(7) + 4(7) + 5(7) + 6(7) = 140$$

- A 70 is ten times seven.
- B 84 is 12 times seven.
- C The last term, $6n$, was missed in the calculation.
- D The first term, $2n$, was missed in the calculation.

2 F

Students should notice the clue word *total* and recognize that it refers to addition. They need to read carefully to see that the problem asks for the total number of coins, and the given variables represent numbers of coins. Addition is all that is required in this expression. Point out that if a reference to coin value is made, multiplication becomes a necessary part of the expression.

$$q + d + n$$

- G Students may have assigned coefficients relative to nickels per coin.
- H Students may have found the value of the coins, in dollars.
- J Students may have separated coin values from the expression for the number of coins.
- K Students may have found the value of the coins in cents.

3. A large jar holds 48 olives, and a small jar holds 32 olives. Which of the following expressions represents the number of olives needed to fill x large jars and y small jars?

A. $48x + 32y$

B. $\frac{x}{48} + \frac{y}{32}$

C. $\frac{xy}{80}$

D. $80xy$

E. $\frac{1,536}{xy}$

hint → Pick numbers for x and y . Solve the problem using those numbers. Then see which answer choice gives the same result.

4. For a room, a hotel charges $55.90d + 35p$ dollars, where d is the number of days the room is rented and p is the number of people staying in the room. What would the hotel charge if a family of 3 people wanted to rent a room for 5 days? (Note: No sales tax is involved.)

F. \$98.90

G. \$202.70

H. \$342.70

J. \$384.50

K. \$727.20

hint → Even though this is a word problem, all you really must do is *Substitute and Compute*.

A

Shared Practice: Answers

3 A

The hint provides one approach, **Picking Numbers**, that students may use. Some students may find it easy enough to merely write an expression for the total number of olives needed, using clue words to assist the process. It is also possible to compare each expression, working backwards to see if it could be written from the given problem. Expressions should be ruled out and eliminated.

$$48x + 32y$$

- B Students may have divided instead of multiplying.
- C Students may have multiplied expressions and divided by the quantities (interchanged steps).
- D Students may have multiplied instead of adding.
- E Students may have multiplied the given numbers and divided by the variables.

4 J

Students should underline the given numbers and their labels. Students can use **Substitute and Compute**, making sure to substitute the numbers into the correct variables, rather than simply following the order they are presented in the problem. The number of days comes first in the expression but second in the given information.

$$55.90(5) + 35(3) = 384.5$$

- F Students may have added the numbers given in the problem and expression.
- G Students may have multiplied 55.90 by the number of people and added 35.
- H Students may have reversed the 3 and 5 and substituted for the incorrect variables.
- K Students may have made a computational error.

Name _____ Date _____

5. The total cost of renting a banquet hall is \$102.00 for each hour the hall is used, plus \$67.50 for each guest who attends. What is the total cost of renting the hall for a 4-hour event with 98 guests?
(Note: No sales tax is involved.)
- A. \$ 678.00
 - B. \$ 6,717.00
 - C. \$7,023.00
 - D. \$ 7,174.00
 - E. \$10,266.00

hint → The clues "total" and "plus" tell you to add. The clue "each" tells you to multiply.

6. If $f(x) = x^3 - x^2 - x$, what is the value of $f(-3)$
- F. -39
 - G. -33
 - H. -21
 - J. -18
 - K. 0

hint → To evaluate $f(-3)$, just substitute -3 for x in $x^3 - x^2 - x$. Then compute.

A

Shared Practice: Answers

5 C

Students should use clue words to write an equation to represent the total cost. Next, students can Substitute and Compute with the given numbers. Let h stand for hours, g stand for number of guests, and C equal the total cost.

$$C = 102h + 67.50g$$

$$C = 102(4) + 67.50(98)$$

$$C = 408 + 6,615$$

$$C = 7,023$$

- A Students may have multiplied each number by 4, the number of hours, and added.
- B Students may have forgotten to multiply the cost per hour by the number of hours.
- D Students may have made a computational error.
- E Students may have reversed the number of hours and the number of guests.

6 G

Students can solve this problem with Substitute and Compute, substituting -3 for x .

$$(-3)(-3)(-3) - (-3)(-3) - (-3) =$$

$$(-27) - (9) + 3 = -36 + 3 = -33$$

- F Students may have subtracted 3 instead of adding.
- H Students may have added 9 instead of subtracting and subtracted 3 instead of adding.
- J Students have added 9 instead of subtracting.
- K Students may have multiplied 9 and 3 instead of adding.

7. Miguel is at a bookstore buying books, each of which usually costs \$12.60. He learns that the books are on sale, and he can buy 5 books for a total of \$55.00. If he buys the books on sale, and the sale price of each book is the same, by what amount is the price of *each* book reduced?
- A. \$ 0.32
 - B. \$ 1.60
 - C. \$ 1.93
 - D. \$ 2.52
 - E. \$ 11.00

hint → Use clue words to help you translate.

8. The cost for a phone call is r cents for the first 3 minutes and s cents for each minute thereafter. What is the cost in cents of a phone call lasting exactly t minutes, where $t > 3$?
- F. $r + st$
 - G. $r + s(t - 3)$
 - H. $3r + st$
 - J. $(3r + s)t$
 - K. $\frac{3r + s}{t}$

hint → Use Picking Numbers to help you solve the problem.
Be sure to pick a number for t that is greater than 3.

A

Shared Practice: Answers

7 B

Students can use clue words to write an equation for the total cost of the books, which is equal to \$55.00. A discount means less than, or subtraction. If \$12.60 equals the regular price of one book, then 12.60 minus d could represent the price after the discount. Students can multiply the price by five to get the equation. Students can then isolate the variable to solve the problem.

$$5(12.60 - d) = 55.00$$

$$63.00 - 5d = 55.00$$

$$-5d = -8$$

$$d = 1.60$$

- A Students may have divided the discount by 5 twice.
- C Students may have made a computational error.
- D Students may have divided the regular cost of one book by 5.
- E Students may have found the cost of one sale book instead of the amount of the discount.

8 G

Students can use clue words to write an equation for the cost of the phone call. There are two logical word groups that must be added to get the cost. The phrase “ s cents for each minute” means multiply. Since s is the cost of each minute after the first 3, students should recognize that they need to multiply s by the number of minutes after the first 3, or t minus 3. Students may also use Picking Numbers to solve the problem or check their work.

$$r + s(t - 3)$$

- F Students may have multiplied s by the whole time, not the time after the first 3 minutes.
- H Students may have confused the variables r and s .
- J Students may not understand the problem or may have guessed.
- K Students may have divided by t instead of multiplying.

Teacher’s Note

When you review these problems, focus on attention to details such as the labels that go with given numbers. Students should reread problems to check for clue words and for accuracy in translating into symbols. They should reread the question to be sure the label on the answer matches what was asked in the question.

Name _____ Date _____

KAP Wrap



Look at the expression below.

$$5a + 6$$

Write a word problem that could be represented by this expression.
Include clue words in your problem to help whoever reads your problem understand how to write an expression for it.

A

Task

Students will reflect on writing and solving algebraic expressions.

Delivery

Commend students for their work with algebra expressions.

Remind students that they can find clue words to help write expressions. They can also use Substitute and Compute to solve or simplify when values for variables are given. If no numbers are given to substitute, remind students they may use Picking Numbers to find an answer.

Preview the KAP Wrap activity with students.

Make sure students understand the directions.

Have students work on this written response at the end of class or for homework.

You may have students share their work with classmates, or you may collect students' writing to assess their developing understanding.