

Name \_\_\_\_\_ Date \_\_\_\_\_

## Slope



### SETTING THE STAGE OPENING ACTIVITY

Two freight trains leave the same loading area at the same time for the same destination. Train A travels 60 miles per hour and Train B travels 80 miles per hour.



- If neither train makes any stops, which one will arrive at the destination first? How do you know?

*Train B will arrive at the destination first because it is traveling*


*at a faster speed than Train A.*

## Behind the Scenes

### Objective

Students will calculate positive, negative, and zero slopes using ordered pairs.

### Setting the Stage

 approx. 5 min.

### Delivery

**Have students complete the Setting the Stage activity independently.**


Alternatively, you may choose to have students complete this activity in pairs.

**Lead a discussion about the slope.**

Have students share their responses to the activity. The discussion should be guided by student responses, but you may wish to address the following points:

- A rate is a ratio that compares quantities with different units. The speed at which each train travels is a rate of change, comparing distance traveled in miles to elapsed time in hours.
- With each passing hour, Train B's distance from Train A always increases by 20 miles.

# Dress Rehearsal

 approx. 15 min.

## Delivery

**Introduce the vocabulary terms for this lesson.**

Read the vocabulary terms and their definitions.

Draw a coordinate plane on the board and lead a review of the vocabulary terms. Have students identify each axis.

Write (3, 8) on the board. Ask students to identify this as either an ordered pair or the coordinates of a point. Have students explain how to plot this point on the coordinate plane.

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### DRESS REHEARSAL INSTRUCTION & GUIDED PRACTICE

#### VOCABULARY TERMS

**Coordinate plane:** A plane formed by a horizontal line (the  $x$ -axis) that intersects a vertical line (the  $y$ -axis)—the plane is divided into four quadrants in which ordered pairs or points are identified and plotted

**Coordinates:** The ordered pair  $(x, y)$  used to represent a point on a coordinate plane

**Ordered pair:** A pair of numbers  $(x, y)$  used to name a point on a coordinate plane

**Plot:** To graph ordered pairs  $(x, y)$  on the coordinate plane

**Point:** A geometric object with position, but neither length nor width

**Slope:** The slant or steepness of a line graphed on the coordinate plane—it is a ratio that compares the change in the  $y$ -values (the rise) to the change in the  $x$ -values (the run); lines that rise up from left to right have positive slopes, and lines that fall down from left to right have negative slopes

**$x$ -axis:** The horizontal axis in a two-dimensional coordinate system

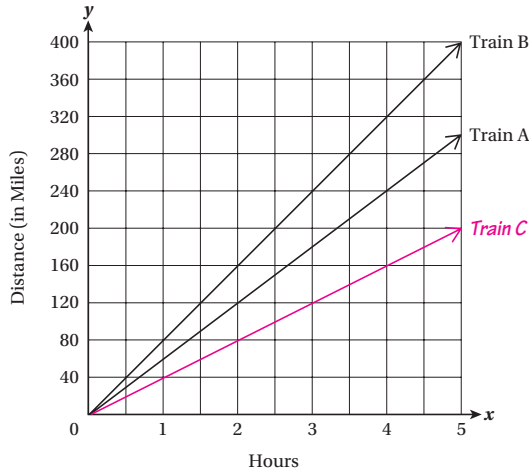
**$y$ -axis:** The vertical axis in a two-dimensional coordinate system



**DRESS REHEARSAL**  
(continued)

**INTRODUCTION**

Train B is the faster train, and each train has the same distance to travel, so Train B will reach the destination first. The total distance each train travels over time can be represented in a graph.



- How many miles does Train A travel in 3 hours? 180 miles
- How many miles does Train B travel in 3 hours? 240 miles

Over any amount of time, Train B covers a greater distance than Train A. The line for Train B is steeper, because the total distance traveled by Train B increases with time at a faster rate than the total distance traveled by Train A.

- Draw a line on the graph above that represents a third train, Train C, which travels more slowly than both Train A and Train B.
- Is the line for Train C more steep or less steep than the lines for Trains A and B, which are faster trains?

The line for Train C is less steep.

# Dress Rehearsal

## Delivery

**Have students relate speed to the steepness of a graph.**

Have students follow along as you read the text aloud. Emphasize that since both trains are starting at the same time and traveling the same distance, the faster train will reach the destination first.

Direct students' attention to the graph, and guide them through the scaffolding questions.

- Discuss the fact that since Train B is faster, it covers a greater distance than Train A over any amount of time. Connect this with the steepness of the lines. Emphasize that the steeper the line, the faster the train.
- Help students understand the meaning of the word "steep." Explain that steeper lines are closer to being vertical, while lines that are less steep are closer to being horizontal. Have students draw a line for a Train C that is slower than both Train A and Train B.

### Teacher's Note

If students are confused, remind them that faster trains have steeper lines. Therefore, a train that is slower than both of the other trains must have a line that is less steep than the other lines. Assure students that any line is correct as long as it begins at zero and is less steep than the other lines.

# Dress Rehearsal

## Delivery

### Explain how to describe steepness using slope.

Read the text aloud, and guide students through the scaffolding questions.

- Explain that a number or fraction can be used to describe the steepness of a line.
- Inform students that to find the slope of a line, they should divide the vertical change by the horizontal change between two points on the line.
- Emphasize that a greater slope indicates a steeper line.
- Clarify that the slope of a line is the same between any two points on that line.

### Teacher's Note

If students are uncertain how slope relates to the graph of a line, remind them that steeper lines have a greater slope. Direct students' attention to the graph representing the two trains in the introduction.

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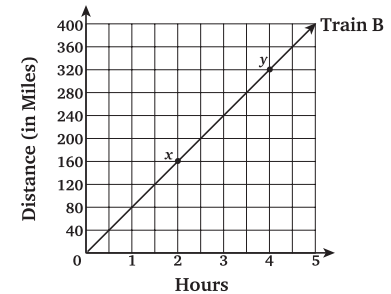


## DRESS REHEARSAL

(continued)

### Describing Steepness using Slope

The steepness of a line can be described numerically with a value known as the **slope** of the line. Slope measures how many units the y-value changes for every unit the x-value changes; therefore, a greater slope indicates a steeper line. To find the slope of a line, pick any two points on the line and divide the vertical change by the horizontal change.



The position of Train B over time is graphed above. What is the slope of the line?

• Find the vertical change between point x and point y.  $320 - 160 = 160$  miles

• Find the horizontal change between point x and point y.  $4 - 2 = 2$  hours

• To determine the slope of the line, fill in the fraction below with the values above.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{160 \text{ miles}}{2 \text{ hours}}$$

• Simplify the fraction.  $80$  miles per hour

• Is the slope of the line representing Train B greater or less than the slope of the line representing Train A? How do you know?

The slope of the line representing Train B is greater because the line is steeper.

In each hour, Train B travels 80 miles. Therefore, the slope of Train B is  $\frac{80}{1}$ , or 80. Because the line is straight, no matter which two points are selected on the line, the ratio of the vertical distance to the horizontal distance will be the same.

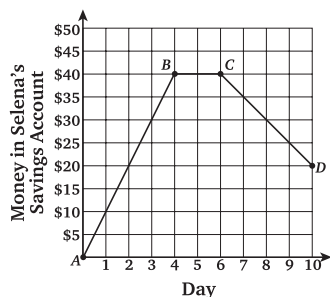


**DRESS REHEARSAL**  
(continued)

**The Slope Formula**

Selena has opened a savings account at a local bank. She deposits 10 dollars a day for the first four days. For the next two days, Selena doesn't make any deposits or withdrawals. Then, for the following four days, Selena withdraws 5 dollars each day.

The graph below represents the money in Selena's savings account over these ten days.



The line segment between point *A* and point *B* describes the account activity over the first four days, during which Selena deposits 10 dollars a day. Since the *y*-axis records the account balance and the *x*-axis records the day, the slope of this line segment describes the change in account balance per day.

Find the slope of the line segment between Point *A* and Point *B*.

- What is the ordered pair for point *A*? (0, 0)
- What is the ordered pair for point *B*? (4, 40)
- What is the difference between the *y*-coordinates?  $40 - 0 = 40$
- What is the difference between the *x*-coordinates?  $4 - 0 = 4$
- Divide the *y*-difference by the *x*-difference to find the slope.  $\frac{40}{4} = 10$
- How is this slope related to Selena's deposits? Explain.

The slope is equal to the amount of money Selena deposits each day, because *y* represents the balance and *x* represents the day.

# Dress Rehearsal

## Delivery

**Explain how to use the slope formula to find the slope of a line.**

Read the text aloud, and guide students through the scaffolding questions.

- Discuss the graph of Selena's bank account activity. Explain that the line between point *A* and point *B* represents Selena's daily deposits, and the line between point *C* and point *D* represents Selena's daily withdrawals.
- Emphasize that, because the vertical direction represents the account balance and the horizontal direction the number of days, the slope of the line segments in the graph describes the change in account balance per day.
- Emphasize that this strategy can always be used to calculate the slope of a line when two ordered pairs on the line are known.
- Guide students through the derivation of the slope formula.

### Teacher's Note

Struggling students may need to be reminded to subtract the *x*- and *y*-coordinates in the same order when calculating the slope.

# Dress Rehearsal

## Delivery

### Point out the possibility that a line has a zero slope.

Read the text aloud, and guide students through the scaffolding questions.

- Remind students that when finding the difference in the coordinates, students should subtract the left-most coordinate from the right-most.
- If students are unsure how to describe the line, emphasize that it is neither rising nor falling. Tell students to think of a word to describe a line that goes directly from left to right without any vertical change.

### Explain that a line can have a negative slope.

Guide students through the scaffolding questions.

- Explain that the slope is negative because the vertical change is negative as the days increase.
- Emphasize that if a line falls from left to right, it has a negative slope.

### Teacher's Note

Students may make the mistake of subtracting the larger coordinate from the smaller coordinate in both cases. Remind students that if they subtract point  $C$  from point  $D$  for one coordinate they must do the same for the other coordinate, even if it yields a negative value.

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### DRESS REHEARSAL

(continued)

If you know the location of two points on a line, you can always find the slope of the line the same way you used above. First, find the difference between the  $y$ -coordinates and the  $x$ -coordinates. Then, divide the  $y$ -difference by the  $x$ -difference.

- If a line includes the ordered pairs  $(x_1, y_1)$  and  $(x_2, y_2)$ , write an equation for the slope of the line.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

The equation you have constructed is known as the *slope formula*.

#### Zero Slope

The line segment between point  $B$  and point  $C$  describes the account activity over the fifth and sixth day, during which Selena does not make any deposits or withdrawals.

Find the slope of the line segment between point  $B$  and point  $C$ .

- What is the ordered pair for point  $B$ ? (4, 40)
- What is the ordered pair for point  $C$ ? (6, 40)
- What is the difference between the  $y$ -coordinates?  $40 - 40 = 0$
- What is the difference between the  $x$ -coordinates?  $6 - 4 = 2$
- Divide the  $y$ -difference by the  $x$ -difference to find the slope.  $\frac{0}{2} = 0$
- What word could you use to describe a line with a slope of zero? horizontal

#### Negative Slope

The line segment between point  $C$  and point  $D$  describes the account activity over the last four days, during which Selena withdraws 5 dollars a day.

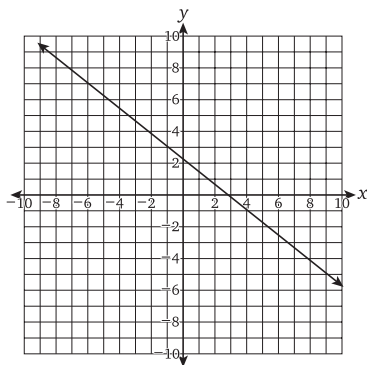
- What is the ordered pair for point  $C$ ? (6, 40)
- What is the ordered pair for point  $D$ ? (10, 20)
- What is the difference between the  $y$ -coordinates?  $20 - 40 = -20$
- What is the difference between the  $x$ -coordinates?  $10 - 6 = 4$
- Divide the  $y$ -difference by the  $x$ -difference to find the slope.  $\frac{-20}{4} = -5$

A negative slope indicates that a line is falling: as the  $x$ -value increases, the  $y$ -value decreases.



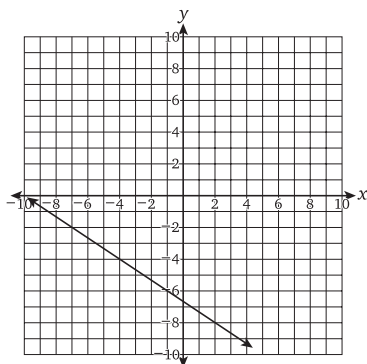
**SHOW TIME**  
INDEPENDENT PRACTICE

- 1) What is the slope of the line shown in the graph below?



$$\frac{0 - 4}{-3 - (-2)} = \frac{-4}{-5} = -\frac{4}{5}$$

- 2) Use the two points on the line below to prove that the slope of the line is  $-\frac{2}{3}$ .



$$\text{slope} = \frac{-8 - (-2)}{-2 - (-7)} = \frac{-6}{9} = -\frac{2}{3}$$

## Show Time

### Delivery

**Have students complete the Show Time activity.**

As students work, observe and assist when necessary. Redirect students as needed by asking them questions about their work. Effective questions include the following:

- What are the coordinates of two points on the line?
- What is the vertical change between the two points? What is the horizontal change?
- Which value goes in the numerator and which value goes in the denominator?

### Teacher's Note

To adjust this activity for struggling students, have students count units on the graph to determine the vertical and horizontal distances rather than using the slope formula. Remind students to pay attention to the positive or negative orientation of the line.

**Reserve a few minutes to review the answers as a class.**

You may wish to discuss each of the questions in this exercise, or you may choose specific questions that were especially challenging for students.

- For question 1, explain that although any two points on the line can be used to calculate slope, the easiest points to use are the  $x$ - and  $y$ -intercepts, if possible.
- For question 2, point out that the fraction  $-\frac{6}{9}$  can be simplified to  $-\frac{2}{3}$ .

# Show Time

## Delivery

### Continue reviewing answers as a class.

You may wish to discuss each of the questions in this exercise, or you may choose specific questions that were especially challenging for students.

- For question 3, remind struggling students that the slope formula calculates rise over run. Coach them to think about which coordinates they would use to calculate the rise and which they would use to calculate the run.
- For question 4, if students are unsure how to draw a line with a slope of  $\frac{1}{2}$ , explain that they can find a second coordinate by moving 1 unit upward and 2 units to the right of the original point.
- For question 5, explain that three separate slopes need to be calculated and compared.

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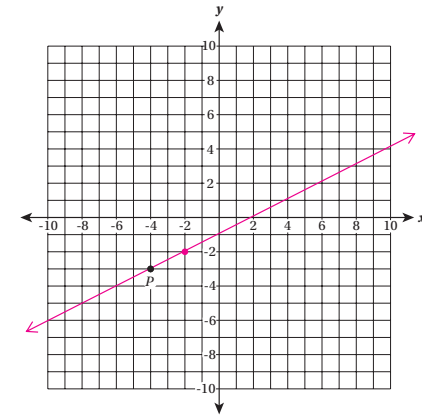


### SHOW TIME (continued)

- 3) The ordered pairs  $(-5, 9)$  and  $(3, -5)$  lie on the same line. What is the slope of this line?

$$\frac{-5 - 9}{3 - (-5)} = \frac{-14}{8} = -\frac{7}{4}$$

- 4) Draw a line below that contains point  $P$  and has a slope of  $\frac{1}{2}$ .



- 5) Use the slope formula to determine if any of the following three ordered pairs lie on the same line.

$(-3, -6)$ ;  $(5, -7)$ ;  $(-11, -9)$

(Hint: If the ordered pairs lie on the same line, the slope between any two ordered pairs is the same.)

$$\text{slope between } (-3, -6) \text{ and } (5, -7): \frac{-7 - (-6)}{5 - (-3)} = \frac{-1}{8} = -\frac{1}{8}$$

$$\text{slope between } (5, -7) \text{ and } (-11, -9): \frac{-9 - (-7)}{-11 - 5} = \frac{-2}{-16} = \frac{1}{8}$$

$$\text{slope between } (-3, -6) \text{ and } (-11, -9): \frac{-9 - (-6)}{-11 - (-3)} = \frac{-3}{-8} = \frac{3}{8}$$

Because the slope is not constant between any of the ordered pairs, none of the pairs lie on the same line.

## Encore! Encore!

### Extension Activities

#### Quick Math

Draw a horizontal line on a coordinate plane. Have students pick two points on the line and use these points to find the slope of the line. Emphasize that the slope of a horizontal line is always zero, since there is no vertical change. Then draw a vertical line and have students attempt to find the slope of this line. Explain that since a number cannot be divided by zero, the slope of a vertical line is undefined.

#### Math Project

Provide students with grid paper and have them draw a side view of any object, large or small. Then have students imagine that the object is the size of a mountain and students need to ride a bicycle up the object. Explain that if a road has a slope greater than  $\frac{1}{5}$ , it is not traversable by car or bicycle. Have students determine whether or not they could ride directly up the object. If not, ask them if they can think of any other ways they could get up the object with a bicycle; encourage them to think about the roads that scale mountains in the real world. Students may recall that mountain roads utilize switchbacks to reduce slope.

#### Reteaching

Explain to students that although the examples of slope used in the lesson all have a  $y$ -variable of time, slope can be used to describe any data represented on a graph. On a coordinate grid, present a side view of a landscape, including a plain and two hills with different slopes, all made up of straight lines. Explain that since the graph is a picture of a physical object, the  $x$ -axis represents horizontal position and the  $y$ -axis represents vertical position.

Have students find the slope of the hills by dividing the change in vertical position by the change in horizontal position. Point out that the steeper hill has a greater slope. Emphasize that the words *slope* and *steepness* can be used to describe both the graph and the actual hills; a larger slope indicates a steeper hill. Explain to students that no matter what is being graphed, they can always think of the line as a hill to help them think about slope.